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The Admissibility of Tests for the Equality of Mean Vectors and Covariance Matrices

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0. Summary

In the previous paper [3], the admissibility of certain tests and classifications in multivariate normal analysis was obtained using the method in Kiefer and Schwartz [2]. In this paper we consider the problem of testing the equality of mean vectors and covariance matrices. Two cases are considered. One case is that of testing the equality of mean vectors and covariance matrices in k normal populations, and the other case is testing the equality of a mean vector and a covariance matrix to given vector and matrix in a normal population. We shall prove admissibility of certain test procedures for the problems by modifying the Kiefer-Schwartz's method. The test procedures include the likelihood ratio test for each problem.

1. Preliminaries

Throughout this paper we consider random matrices whose columns are independently distributed, each p-variate normal. The parameter space in each problem will be denoted by $\Omega=\{\theta\}=H_0+H_1$. The Lebesgue density function of X for given θ will be denoted by $f_X(x;\theta)$. *A priori* probability measure or its constant multiples will be denoted by Π and $\Pi=\Pi_0+\Pi_1$ with Π_i a finite measure on H_i.

Let $V=(X, U)$ be a random matrix whose columns, under H_1, have common unknown covariance matrix Σ and $\mathrm{E}U=\nu(p\times 1)$ (unspecified). Let θ^* be the parameter of the distribution of X, i.e., $\theta=(\theta^*, \nu)$. Let H_1^* be the domain of θ^* under H_1, and consider the case where the domain of ν is E^p and $H_1=H_1^*\times\mathrm{E}^p$, i.e., $\theta\in H_1$ if and only if $\theta^*\in H_1^*$. Let H_1^{**} be a subset of H_1^* for which Σ can be written as $\Sigma=(C_0+D)^{-1}$ where C_0 is a given positive definite matrix and D is nonnegative definite matrix. And consider a finite measure Π_1^* on H_1^* which assigns whole measure to H_1^{**}. Then the following lemma holds:

LEMMA 1.1. *There exists a finite measure Π_1 on H_1 which satisfies*

$$\int f_V(v;\theta)\,\Pi_1(d\theta)=c\cdot\mathrm{etr}\left\{-\frac{1}{2}C_0(U-\nu_0)(U-\nu_0)'\right\}\cdot\int f_X(x;\theta^*)\Pi_1^*(d\theta^*) \tag{1.1}$$

for any fixed vector ν_0.