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Nonimbedding Theorems of Lie Algebras

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§ 1.

Let N be a Lie or associative algebra, and $\mathfrak{D}$ a set of derivations of N which contains all the inner derivations. Two sequences of subspaces $\{N\mathfrak{D}^i\}$ and $\{N\mathfrak{D}_i\}$ are defined inductively as follows ([2, 3]):

$$N\mathfrak{D}^i = N\mathfrak{D}^{i-1}\mathfrak{D}$$

$$N\mathfrak{D}_i = \{x \in N;\ x\mathfrak{D} \subseteq N\mathfrak{D}_{i-1}\},$$

where $N\mathfrak{D}^0 = N$ and $N\mathfrak{D}_0 = 0$. N is called *$\mathfrak{D}$-nilpotent of class n*, when $N\mathfrak{D}^{n-1} \neq 0$ and $N\mathfrak{D}^n = 0$.

Several authors have investigated the nonimbedding of nilpotent algebras. Namely, Chao [1] showed that a non-abelian Lie algebra A such that its center is 1-dimensional or dim $A/[A, A]=2$ cannot be any $N\mathfrak{J}^i$, where $\mathfrak{J}$ is the algebra of all inner derivations of a nilpotent Lie algebra N. Ravisankar [2] improved this result as follows: Such an algebra A cannot be any $N\mathfrak{D}^i$ of a $\mathfrak{D}$-nilpotent algebra N. Moreover, Tôgô and Maruo [3] proved the following theorems among other things:

Let N be a $\mathfrak{D}$-nilpotent algebra, and A a non-abelian subalgebra of N.

(a) If dim $A/[A, A]=2$, then it is impossible that

$$N\mathfrak{D}^i \supseteq A \supseteq N\mathfrak{D}^{i+1} \qquad (i \geq 1).$$

(b) If A is mapped into $[A, A]$ by every derivation of A, then it is impossible that

$$N\mathfrak{D}^i \supseteq A \supseteq N\mathfrak{D}^{i+1} \qquad (i \geq 1) \quad \text{if } A \text{ is Lie,}$$

$$A = N\mathfrak{D}^1 \text{ or } N\mathfrak{D}^i \supseteq A \supseteq N\mathfrak{D}^{i+1} \qquad (i \geq 2) \quad \text{if } A \text{ is associative.}$$

The purpose of the present paper is to improve these two results about nonimbedding of algebras. Hereafter, we suppose that N is a Lie or associative algebra, $\mathfrak{J}(N)$ is the Lie algebra of all inner derivations of N, $\mathfrak{D}$ is a subset of the derivation algebra of N which contains $\mathfrak{J}(N)$, and N is $\mathfrak{D}$-nilpotent of class n.

We shall need the following result stated in [3].

LEMMA. *The sequences $\{N\mathfrak{D}^i\}$ and $\{N\mathfrak{D}_i\}$ are monotone decreasing and*