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An Integral Representation of an Eigenfunction of the Laplacian on the Euclidean Space

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§ 1. Introduction

The classical theory about Dirichlet problem shows that certain classes of harmonic functions on the unit disk are given by the Poisson integral (cf. [1]). However, as Helgason proved in [4], to obtain arbitrary harmonic functions one has to consider the Poisson integral of "hyperfunctions". He also proved that any eigenfunction of the laplacian (with respect to the Poincaré metric) can be given by the Poisson integral of hyperfunctions.

The present paper deals with the similar problem about eigenfunctions of the laplacian on the n-dimensional euclidean space.

Suggested by the work of Ehrenpreis [2], we define the map $\mathscr{P}_\lambda$ which is an analogue of the Poisson integral [see § 4].

In our case, contrary to the usual Poisson integral, it is not sufficient to consider the hyperfunctions to obtain arbitrary eigenfunctions of the laplacian, but one should consider a certain space $\tilde{\mathscr{B}}(S^{n-1})$ which contains the space of hyperfunctions on the $(n-1)$-dimensional unit sphere as a proper subspace. We shall prove in § 5 that our map $\mathscr{P}_\lambda$ gives an isomorphism of $\tilde{\mathscr{B}}(S^{n-1})$ onto the space of the eigenfunctions of the laplacian.

In this paper we deal with the case where $\lambda \neq 0$. We shall discuss the case where $\lambda = 0$ in the forthcoming paper [5].

§ 2. Review of the repesentation theory of $SO(n)$

In this section we summarize briefly the representation theory of $\boldsymbol{SO}(n)$. $\boldsymbol{SO}(n)$ acts on $\boldsymbol{R}^n$ and if we denote by H the isotropy subgroup of $\boldsymbol{SO}(n)$ at ${}^t(1, 0, \cdots, 0) = e_1$ in $\boldsymbol{R}^n$, then H consists of all elements of the form

$$\begin{pmatrix} 1 & 0 \\ 0 & h \end{pmatrix} \qquad (h \in \boldsymbol{SO}(n-1)).$$

The orbit of e_1 of $\boldsymbol{SO}(n)$ is canonically isomorphic to S^{n-1}, the unit sphere in $\boldsymbol{R}^n$. So we obtain an isomorphism

$$S^{n-1} \ni \omega = g \cdot e_1 \longleftrightarrow gH \in \boldsymbol{SO}(n)/H$$