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Polarizations of Certain Homogeneous Spaces and Most Continuous Principal Series

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§ 1. Introduction

Our main purpose in this paper is to construct unitary representations of the most continuous principal series, using polarizations. As is stated in §1 of [13], a polarization on a symplectic manifold was devised by Kostant with the aim of constructing unitary representations for an arbitrary Lie group. It is an extension of the nilpotent case given in Kirillov [8], and has enough effectiveness in solvable Lie groups of type I (Auslander-Kostant [2]). For semisimple Lie groups, however, the situation is slightly different from them. For example, it has been pointed out by many people that the discrete series representations of a non-compact semisimple Lie group of the non-Hermitian type can not be obtained by polarizations only, and some concepts, like cohomology spaces, seem to be required. However, we can show that the representations of the most continuous principal series can all be constructed by using polarizations (Theorem 6.6). This is partly because a polarization of any semisimple element in the Cartan subalgebra with maximal vector part can be chosen related with a minimal parabolic subalgebra by translating the element by the addition of a certain nilpotent element, and partly because the differential equations attached to the polarization can be replaced by the Borel-Weil theorem of a compact reductive Lie group. In this paper, we also make investigations in each simple Lie algebra, and prove that in case of (AI-AIII), $\mathfrak{so}(n, 1)$ or (EIV), every element has w-polarizations, while there exists an element with no polarizations in Lie algebras of any other type (Theorem 4.6). The proof is made by using a suitable TDS with high singularity.

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§ 2. Real admissible polarizations

In this paper, except for §5, we assume that G is a connected real semisimple Lie group with Lie algebra $\mathfrak{g}_R$. (In §3, $\mathfrak{g}_R$ is assumed to be simple.) Let $\mathfrak{g}$ be the complexification of $\mathfrak{g}_R$, and B the Killing form of $\mathfrak{g}$. Notations