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On the m-Accretiveness of Nonlinear Operators in Banach Spaces

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Introduction

In the theory of nonlinear contraction semigroups, the notion of accretive operators was introduced as a generalization of the notion of the infinitesimal generators, and studied by many authors (see e.g., [1], [2], [3], [4], [6], [8], [10], [11]).

In the present paper we study a multivalued accretive operator A from a Banach space X into itself. It is called m-accretive if the range of $I+A$ is the whole of X. The studies on the m-accretiveness of nonlinear operators were made by T. Kato [6], R.H. Martin, Jr. [9], G.F. Webb [12], the author [7] and others. The purpose of this paper is to give a necessary and sufficient condition for m-accretiveness; under certain conditions, an accretive operator A from X into X is m-accretive if and only if it is demiclosed and the initial value problem

$$\begin{cases} \dfrac{du(t)}{dt} + Au(t) \ni z \\ u(0) = x \end{cases}$$

has a solution (in a certain sense) on $[0, \infty)$ for each $x \in D(A)$ and $z \in X$ (THEOREM 1). It was announced by F.E. Browder [2] that if the dual space of X is uniformly convex, then a densely defined singlevalued accretive operator A is m-accretive if and only if $-(A+z)$ is the weak infinitesimal generator of a nonlinear contraction semigroup on X for each $z \in X$. This was proved by M.G. Crandall and A. Pazy [4] in case X is a Hilbert space. In this paper we shall prove Browder's announcement in a more general form, namely, when A is multivalued.

§1. Definitions and notation

Throughout this paper let X be a real Banach space and X^* be its dual space. The natural pairing between $x \in X$ and $x^* \in X^*$ is denoted by $<x, x^*>$. The norms in X and X^* are denoted by $\|\cdot\|$ and the identity mapping in X by