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Groups of Self-equivalences of Certain Complexes

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Introduction

Throughout this note, all spaces, maps and homotopies are assumed to be based, and any map and its homotopy class are written by the same letter.

Let $\mathscr{E}(X)$ denote the group of self-equivalences of a topological space X. The member of $\mathscr{E}(X)$ is a homotopy class of homotopy equivalences of X into itself. The group operation of $\mathscr{E}(X)$ is given by the composition of maps. This group $\mathscr{E}(X)$ is a homotopy type invariant of X.

Several examples are known (see [5]-[10]). In particular, for a CW-complex $K=S^n\cup e^{n+k+1}$, $k\geqq -1$, having two cells, the group $\mathscr{E}(K)$ has been studied in the case $k=-1$, $n\geqq 2$ and the case $k=0$, $n\geqq 1$. The former case is treated in [9: Example 8], and the latter is due to P.Olum [7] for $n=1$ and the recent work of A.J. Sieradski [10] for arbitrary $n\geqq 1$.

The purpose of this note is to determine the group $\mathscr{E}(K)$ for a CW-complex $K=S^n\cup_\alpha e^{n+k+1}$, $k\geqq 1$, under the condition that the attaching class α is a double suspension, $\alpha=E^2\alpha''$, and both α and $E\alpha''$ have the same order. Our main result is stated as follows:

THEOREM 3.2. *Let $K=S^n\cup_\alpha e^{n+k+1}$, $k\geqq 1$, $n\geqq 2$. Suppose that there exists an element $\alpha''\in\pi_{n+k-2}(S^{n-2})$ such that $E^2\alpha''=\alpha$, and both $E\alpha''$ and α have the same order m. Let $i: S^n\to K$ and $p: K\to S^{n+k+1}$ be the inclusion and the projection, respectively, and set*

$$G=i_*p^*\pi_{n+k+1}(S^n),$$

which is a subgroup of the group $[K, K]$ with the track addition.

Define a two-sided action of the multiplicative group $Z_2=\{-1, 1\}$ on G by

$$(-1)g=i_*p^*(-\iota_n)\gamma,\quad g(-1)=-g\quad \textit{for}\quad g=i_*p^*\gamma\in G,$$

where $\iota_n\in\pi_n(S^n)$ is the class of the identity map of S^n.

Then, the group $\mathscr{E}(K)$ of self-equivalences of K is isomorphic to the multiplicative group whose entries are matrices

$$\begin{pmatrix} x & g\\ 0 & y\end{pmatrix},\qquad x,\ y\in Z_2,\quad g\in G\qquad\qquad \textit{for } m=1, 2,$$

$$\begin{pmatrix} x & g\\ 0 & x\end{pmatrix},\qquad x\in Z_2,\quad g\in G,\qquad\qquad \textit{for } m>2,$$

where the matrix multiplication is given as usual.