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# *Energy of Functions on a Self-adjoint Harmonic Space II*

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## Introduction

In the previous paper [13] under the same title, we introduced a notion of energy of functions on a self-adjoint harmonic space. By a self-adjoint harmonic space, we mean a Brelot's harmonic space possessing a symmetric Green function. We showed that a notion of energy which is given in terms of differentiation in the classical case can be defined on such an abstract harmonic space. In [13], however, we defined energy only for certain bounded functions and for harmonic functions. In the present paper, we shall extend the definition to more general functions, which correspond to BLD-functions (see [10] and [5]) or Dirichlet functions (see [9]) in the classical potential theory.

Here, let us review basic definitions and main results in [13].

The base space $\Omega$ is a connected, locally connected, noncompact, locally compact Hausdorff space with a countable base. We consider a structure of harmonic space $\mathfrak{H}=\{\mathscr{H}(\omega)\}_{\omega:\mathrm{open}\subset\Omega}$ on $\Omega$ satisfying Axioms 1, 2 and 3 of M. Brelot [4]. In addition to these axioms, we assume:

*Axiom* 4. The constant function 1 is superharmonic.

*Axiom* 5. There exists a positive potential on $\Omega$.

*Axiom* 6. Two positive potentials with the same point (harmonic) support are proportional.

The pair $(\Omega, \mathfrak{H})$ is called a *self-adjoint harmonic space* if there exists a function $G(x, y)$: $\Omega\times\Omega\to(0, +\infty]$ such that $G(x, y)=G(y, x)$ for all $x, y\in\Omega$ and, for each $y\in\Omega$, $x\to G(x, y)$ is a potential on $\Omega$ and is harmonic on $\Omega-\{y\}$. Such $G(x, y)$ is uniquely determined up to a multiplicative constant and is called a *Green function* for $(\Omega, \mathfrak{H})$. In our theory, we assume that $(\Omega, \mathfrak{H})$ is a self-adjoint harmonic space and fix a Green function $G(x, y)$ throughout. For any domain $\omega$ in $\Omega$, $\mathfrak{H}|\omega=\{\mathscr{H}(\omega')\}_{\omega'\subset\omega}$ is also a structure of self-adjoint harmonic space on $\omega$ satisfying Axioms $1\sim6$ and there is a Green function $G^{\omega}(x, y)$ for $(\omega, \mathfrak{H}|\omega)$ having the same singularity as $G(x, y)$ (see Proposition 1.2). For a non-negative measure (=Radon measure) $\mu$ on $\Omega$ (resp. on $\omega$) $U^{\mu}(x)=\int_{\Omega}G(x, y)d\mu(y)$ (resp.