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An Independence Condition in Semi-Infinite Programs

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§1. Introduction

An independence condition was introduced by M. Ohtsuka [2] in relation with the condititonal Gauss variational problem. This notion was generalized by the author [3] and applied to the study of semi-infinite programs. It was shown in [3] that a decomposition theorem (Lemma 4 in [3]), i.e., the existence of a full system of components, plays an important role in the study of the conditional Gauss variational problem. One of our aims is to further generalize the independence condition and the decomposition theorem. By making use of our decomposition theorem, we shall study a change of values of semi-infinite programs.

§2. An independence condition

Denote by R^n the n-dimensional Euclidean space, by R_0^n the positive orthant of R^n and by e_k the vector in R^n whose j-th coordinate is equal to 0 if $j \neq k$ and 1 if $j=k$. We set $R=R^1$ and $R_0=R_0^1$. For a subset B of R^n, we denote by B° the interior of B in R^n. Denote by $((v, w))$ and $\|w\|$ the usual inner product of v, $w \in R^n$ and the usual distance from 0 to $w \in R^n$ respectively, i.e.,

$$((v, w)) = \sum_{j=1}^{n} r_j s_j \quad \text{and} \quad \|w\| = [((w, w))]^{1/2}$$

for $v=(r_1, \ldots, r_n)$ and $w=(s_1, \ldots, s_n)$.

Let X be a real linear space and P be a convex subset of X such that $0 \in P$. Let $f_i(x)$ $(i=1, \ldots, n)$ be a real-valued function defined on P satisfying the following conditions:

(a) $f_i(tx)=tf_i(x)$ for all $t \in R_0$ and $x \in P$ such that $tx \in P$,

(b) $f_i(x+y)=f_i(x)+f_i(y)$ for all x, $y \in P$ such that $x+y \in P$.

In case P is a convex cone, conditions (a) and (b) imply that $f_i(x)$ is positively homogeneous and additive.

Now we introduce an independence condition which coincides with the one in [3] in the case where P is a convex cone.

DEFINITION. Let $x \in P$. We say that $\{f_i\}=\{f_i; i=1, \ldots, n\}$ is x-indepen-