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On the Irreducibility of Induced Representations of SU (2, 1)

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§ 1. Introduction

This note is concerned with the irreducibility of representations of $SU(2, 1)$ induced from one-dimensional representations of its minimal parabolic subgroup. Let $B=MA_+N$ be the minimal parabolic subgroup of G associated with an Iwasawa decomposition KA_+N of the group $G=SU(2, 1)$. Let $\mathfrak{g}_0=\mathfrak{su}(2, 1)$ be the Lie algebra of G, and $\mathfrak{a}_+$ (resp. $\mathfrak{n}_0$) the subalgebra of $\mathfrak{g}_0$ corresponding to A_+ (resp. N), and we define a linear form ρ on $\mathfrak{a}_+$ by

$$\rho(H)=2^{-1}\mathrm{Trace}(ad_{\mathfrak{n}_0}(H))$$

for every $H\in\mathfrak{a}_+$. Then a unitary character σ of M and a complex number λ define a representation $\mu_{\sigma\lambda}$ of B by

$$\mu_{\sigma\lambda}(m(\exp H)n)=\sigma(m)\exp(\lambda\rho(H))$$

for $m\in M$, $H\in\mathfrak{a}_+$ and $n\in N$. Let $\tilde{X}^{\sigma\lambda}$ be the space of all $\boldsymbol{C}$-valued C^∞-differentiable functions f on G such that

$$f(xb)=\mu_{\sigma,\lambda+1}(b^{-1})f(x)$$

for every $x\in G$ and $b\in B$. The group G acts on $\tilde{X}^{\sigma\lambda}$ by left-translations, and there exists a canonical G-invariant non-singular pairing between $\tilde{X}^{\sigma\lambda}$ and $\tilde{X}^{\sigma,-\bar{\lambda}}$. The universal enveloping algebra $\mathfrak{U}$ of the complexification $\mathfrak{g}$ of $\mathfrak{g}_0$ acts on $\tilde{X}^{\sigma\lambda}$ as infinitesimal representations of left-translations, and stabilizes the subspace $X^{\sigma\lambda}$ of $\tilde{X}^{\sigma\lambda}$ consisting of all K-finite elements. The K-module $X^{\sigma\lambda}$ has the irreducible decomposition

$$X^{\sigma\lambda}=\bigoplus_{\tau\in E_K^\sigma} X_\tau^{\sigma\lambda}$$

where E_K^σ is the set of all equivalence classes of irreducible unitary representations of K which contain σ when restricted to the subgroup M, and $X_\tau^{\sigma\lambda}$ denotes the K-submodule of $X^{\sigma\lambda}$ equivalent to τ. We shall make investigations into the irreducibility of the $\mathfrak{U}$-module $X^{\sigma\lambda}$ by using its K-module structure and a canonical pairing (,) of $X^{\sigma\lambda}$ and $X^{\sigma,-\bar{\lambda}}$ The set E_K^σ contains a one-dimensional