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Oscillation and Asymptotic Behavior of Solutions of Retarded Differential Equations of Arbitrary Order

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1. Introduction

We are here concerned with the oscillatory behavior of solutions of higher-order retarded differential equations of the form

$$\text{(A)} \qquad y^{(n)}(x)+y(g(x))F([y(g(x))]^2, x)=0, \qquad n \geqq 2,$$

where the following conditions are always assumed to hold:

(a) $g(x)$ is continuous for $x>0$, $g(x) \leqq x$ and $\lim_{x\to\infty} g(x)=\infty$;

(b) $yF(y^2, x)$ is continuous for $x>0$ and $|y|<\infty$, and $F(t, x)$ is nonnegative for $t \geqq 0$ and $x>0$.

Equation (A) is classified according to the nonlinearity of $F(t, x)$ with respect to t, namely (A) is called *superlinear* if F satisfies

$$\text{(1.1)} \qquad F(t_1, x) \leqq F(t_2, x), \quad t_1<t_2, \quad x \in (0, \infty),$$

and *sublinear* if F satisfies

$$\text{(1.2)} \qquad F(t_1, x) \geqq F(t_2, x), \quad t_1<t_2, \quad x \in (0, \infty).$$

Moreover, (A) is called *strongly superlinear* if there is an $\varepsilon>0$ such that

$$\text{(1.3)} \qquad t_1^{-\varepsilon}F(t_1, x) \leqq t_2^{-\varepsilon}F(t_2, x), \quad t_1<t_2, \quad x \in (0, \infty),$$

and *strongly sublinear* if there is an $\varepsilon>0$ such that

$$\text{(1.4)} \qquad t_1^{\varepsilon}F(t_1, x) \geqq t_2^{\varepsilon}F(t_2, x), \quad t_1<t_2, \quad x \in (0, \infty).$$

(See e.g. Nehari [29], Coffman and Wong [8].) The prototype of equation (A) is

$$\text{(B)} \qquad y^{(n)}(x)+p(x)|y(g(x))|^{\alpha}\operatorname{sgn} y(g(x))=0,$$

where $p(x) \geqq 0$ for $x>0$ and $\alpha>0$, which may be considered as a generalization of the Emden-Fowler equation. Equation (B) is superlinear, strongly superlinear, sublinear or strongly sublinear according as $\alpha \geqq 1$, $\alpha>1$, $\alpha \leqq 1$ or $\alpha<1$.