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On the K-Ring of S^{4n+3}/H_m

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§ 1. Introduction

The purpose of this note is to study the K-ring $K(N^n(m))$ of complex vector bundles over the $(4n+3)$-dimensional quotient manifold

$$N^n(m)=S^{4n+3}/H_m, \qquad (m\geqq 2).$$

Here, H_m is the generalized quaternion group generated by the two elements x and y with the two relations

$$x^{2^{m-1}}=y^2 \quad \text{and} \quad xyx=y,$$

that is, H_m is the subgroup of the unit sphere S^3 in the quaternion field $\boldsymbol{H}$ generated by the two elements

$$x=\exp(\pi i/2^{m-1}) \quad \text{and} \quad y=j,$$

and the action of H_m on the unit sphere S^{4n+3} in the quaternion $(n+1)$-space $\boldsymbol{H}^{n+1}$ is given by the diagonal action.

Recently, the problem of immersing or embedding this manifold $N^n(m)$ in euclidean spaces is studied in [8].

Let α' and β' be the complex line bundles over $N^n(m)$ whose first Chern classes are the generators of $H^2(N^n(m); Z)=Z_2\oplus Z_2$, and $\delta'=\pi^!\lambda$ be the complex plane bundle over $N^n(m)$ induced from the canonical complex plane bundle λ over the quaternion projective space HP^n by the natural projection

$$\pi: N^n(m)\longrightarrow HP^n.$$

Then we have the following

THEOREM 1.1. *The reduced K-ring $\tilde{K}(N^n(m))$ $(m\geqq 2)$ is generated multiplicatively by the three elements*

$$\alpha=\alpha'-1, \quad \beta=\beta'-1 \quad \textit{and} \quad \delta=\delta'-2.$$

This theorem shows that the natural ring homomorphism

$$\xi: \tilde{R}(H_m)\longrightarrow \tilde{K}(N^n(m))$$