

The Fourier Transform of the Schwartz Space on a Semisimple Lie Group

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1. Introduction

Let G be a semisimple Lie group and $L^2(G)$ denote the space of square-integrable functions on G with respect to the Haar measure. The Fourier transform \mathcal{F} can be regarded as an isometry of $L^2(G)$ onto the Hilbert space $L^2(\hat{G})$ which is defined by irreducible unitary representations of G .

In his paper [6(m)], Harish-Chandra introduces the Schwartz space $\mathcal{C}(G)$ consisting of functions on G . It is analogous to the Schwartz space $\mathcal{S}(\mathbf{R}^n)$ of rapidly decreasing functions on a euclidean space \mathbf{R}^n and is contained densely in $L^2(G)$. It is of much interest to ask about the image of $\mathcal{C}(G)$ in $L^2(\hat{G})$ under \mathcal{F} . This is a Paley-Wiener type question for $\mathcal{C}(G)$. There are some results for this problem. It is solved by J.G. Arthur[1] in the real rank one case. Moreover, the problems for the Schwartz space on Riemannian globally symmetric spaces and for a certain subspace are studied by Eguchi-Okamoto[4] and Harish-