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The Fourier Transform of the Schwartz Space on a Semisimple Lie Group

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Table of Contents

1. Introduction

Let G be a semisimple Lie group and $L^2(G)$ denote the space of square-integrable functions on G with respect to the Haar measure. The Fourier transform $\mathscr{F}$ can be regarded as an isometry of $L^2(G)$ onto the Hilbert space $L^2(\hat{G})$ which is defined by irreducible unitary representations of G.

In his paper [6(m)], Harish-Chandra introduces the Schwartz space $\mathscr{C}(G)$ consisting of functions on G. It is analogous to the Schwartz space $\mathscr{S}(\boldsymbol{R}^n)$ of rapidly decreasing functions on a eulidean space $\boldsymbol{R}^n$ and is contained densely in $L^2(G)$. It is of much interest to ask about the image of $\mathscr{C}(G)$ in $L^2(\hat{G})$ under $\mathscr{F}$. This is a Paley-Wiener type question for $\mathscr{C}(G)$. There are some results for this problem. It is solved by J.G. Arthur[1] in the real rank one case. Moreover, the problems for the Schwartz space on Riemannian globally symmetric spaces and for a certain subspace are studied by Eguchi-Okamoto[4] and Harish-