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On the Group of Self-Equivalences of a Mapping Cone

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Introduction

The set $\mathscr{E}(X)$ of homotopy classes of self-(homotopy-)equivalences of a based space X forms a group by the composition of maps, and this group is studied by several authors.

The purpose of this note is to study the group $\mathscr{E}(C_f)$ for a mapping cone $C_f = B \cup_f CA$ of $f: A \to B$ with certain conditions, by the dual considerations of J. W. Rutter [11] using the homotopy exact sequences of cofiberings.

In §1, after preparing some results on $\mathscr{E}(C_f)$, we represent the group $\mathscr{E}(B \vee SA)$, which is the case that f is the constant map, as the split extension of a certain group H by $\mathscr{E}(B) \times \mathscr{E}(SA)$ (Theorem 1.13). In the case that A is the $(m-1)$-sphere S^{m-1}, the above group H is equal to the homotopy group $\pi_m(B)$.

In §2, we have the exact sequence

$$0 \longrightarrow H \longrightarrow \mathscr{E}(B \cup_f e^m) \longrightarrow G \longrightarrow 1$$

for $A = S^{m-1}$, where H is the factor group of $\pi_m(B)$ and G is the subgroup of $\mathscr{E}(B) \times \mathscr{E}(S^m) = \mathscr{E}(B) \times Z_2$. This result is essentially the theorem of W. D. Barcus and M. G. Barratt [1, Th. 6.1].

Furthermore, we study in §3 some cases that the above sequence is split. For the case $2f = 0$, we see in Theorem 3.9 that G is the direct product $G_1 \times G_2$ and the subgroup $G_2 = 1 \times Z_2$ is split. By these results, we obtain in Theorem 3.13 the split extension

$$0 \longrightarrow H \longrightarrow \mathscr{E}(S^n \cup_f e^m) \longrightarrow G \longrightarrow 1$$

for a two-cell complex $S^n \cup_f e^m$ $(2 \leqq n \leqq m-2)$ whose attaching map $f \in \pi_{m-1}(S^n)$ is a suspension Sf' and the orders of f and f' are equal. Here

$$H = \pi_m(S^n)/(f_* \pi_m(S^{m-1}) + (Sf)^* \pi_{n+1}(S^n)),$$

$$G = Z_2 \times Z_2 \quad \text{if} \quad 2f = 0, \qquad = Z_2 \quad \text{if} \quad 2f \neq 0,$$

and the action of G on H is given by

$$(\tau, \rho) \cdot a = \tau a \rho \quad \text{for} \quad a \in \pi_m(S^n),\ (\tau, \rho) \in Z_2 \times Z_2, \quad \text{if} \quad 2f = 0,$$

$$\rho \cdot a = a \rho \quad \text{for} \quad a \in \pi_m(S^n),\ \rho \in Z_2, \quad \text{if} \quad 2f \neq 0.$$