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Oscillations of Differential Equations with Retardations

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This paper is concerned with the oscillatory and asymptotic behavior of the n-th order $(n>1)$ differential equation with retarded arguments

$$(*) \qquad x^{(n)}(t)+f(t, x[g_1(t)], x[g_2(t)], \dots, x[g_m(t)])=0$$

where the functions g_i, $i=1, 2, \dots, m$ are differentiable on the half line $[t_0, \infty)$ and such that

(I) $g_i(t)\leqq t$ for every $t\geqq t_0$
(II) $g_i'(t)\geqq 0$ for every $t\geqq t_0$
(III) $\lim_{t\to\infty} g_i(t)=\infty$

Our results extend previous ones concerning retarded differential equations of the form

$$x^{(n)}(t)+f(t, x[g(t)])=0$$

(Cf. [8] and [2]). Moreover, the results given here can be used in order to obtain other ones concerning retarded differential equations of a more general form than $(*)$, i.e., when f depends on the derivatives too. This can be done by the comparison principle introduced by the authors in [9] and [10]. Thus, recent related results given by Onose [5] and Kusano and Onose [2] could be improved.

In what follows we consider only solutions of $(*)$ which are defined for all large t. The oscillatory character is considered in the usual sense, i.e., a solution of $(*)$ is called *oscillatory* if it has no last zero, otherwise it is called *nonoscillatory*.

To obtain our results we need the following three lemmas, the first of which is an adaptation of a lemma due to Kiguradze [1] and the others of lemmas in [7] and [9].

LEMMA 1. *If u is an n-times differentiable function on $[a, \infty)$ with $u^{(k)}$, $k=0, 1, \dots, n-1$, absolutely continuous on $[a, \infty)$ and if*

$$u(t)\neq 0 \text{ and } u(t)u^{(n)}(t)\leqq 0 \qquad \text{for every } t\in[a, \infty)$$

then there exists an integer l with $0\leqq l<n$, $n+l$ odd and such that

$$u(t)u^{(k)}(t)\geqq 0 \text{ for every } t\in[a, \infty) \qquad (k=0, 1, \dots, l)$$