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On the KO-Ring of S^{4n+3}/H_m

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§1. Introduction

The purpose of this note is to study the KO-ring $KO(N^n(m))$ of real vector bundles over the $(4n+3)$-dimensional quotient manifold

$$N^n(m) = S^{4n+3}/H_m \qquad (m \geqq 2),$$

whose K-ring $K(N^n(m))$ of complex vector bundles is studied in the previous note [3]. Here, H_m is the generalized quaternion group generated by two elements x and y with the two relations

$$x^{2^{m-1}} = y^2 \quad \text{and} \quad xyx = y,$$

that is, H_m is the subgroup of the unit sphere S^3 in the quaternion field $\boldsymbol{H}$ generated by the two elements

$$x = \exp(\pi i/2^{m-1}) \quad \text{and} \quad y = j,$$

and the action of H_m on the unit sphere S^{4n+3} in the quaternion $(n+1)$-space $\boldsymbol{H}^{n+1}$ is given by the diagonal action.

Consider the real line bundles

$$\alpha'_0, \quad \beta'_0 \in KO(N^n(m)),$$

whose first Stiefel-Whitney classes generate the cohomology group $H^1(N^n(m); Z_2) = Z_2 \oplus Z_2$, and the real restriction

$$\delta'_0 = r\pi^! \lambda \in KO(N^n(m))$$

of the induced bundle $\pi^!\lambda$, where λ is the canonical complex plane bundle over the quaternion projective space $HP^n = S^{4n+3}/S^3$ and $\pi: N^n(m) \to HP^n$ is the natural projection. Also, it is proved by B. J. Sanderson [7] that the complexification $c: KO(HP^n) \to K(HP^n)$ is monomorphic and $(\lambda-2)^2 \in cKO(HP^n)$, and so we can consider the element

$$x_0 = \pi^! c^{-1}((\lambda-2)^2) \in KO(N^n(m)).$$

Then we have the following