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*A Remark on Certain Symmetric Stable Processes**

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1. In a series of author's works [2], [3], etc., we try to clarify the relation between the fine topology and the order of divergence of Green functions corresponding to Markov processes. The main result is as follows. There are given Markov processes X_i, $i=1, 2$ on the same "good" state space E which have Green functions $G_i(x, y)$, $i=1, 2$ respectively. If

$$G_1(x, y) \approx G_2(x, y),^{1)}$$

then the fine topologies given by X_i, $i=1, 2$ are equivalent under certain regularity conditions on X_i, $i=1, 2$. Moreover it is shown that a certain order relation of the divergence at the diagonal of $G_i(x, y)$, $i=1, 2$, induces a relation on strength of the fine topologies given by X_i, $i=1, 2$, [2].

In this note we show by examples that

$$G_1(x, y) \leqq \text{Const.}\, G_2(x, y)$$

does not always imply that the fine topology induced by X_2 is stronger than that induced by X_1[2]. In addition our examples show that the fine topologies are not equivalent and the orders of divergence of Green functions are different, even if polar sets (sets of capacity zero) corresponding to X_i, $i=1, 2$ coincide[3].

2. Let X be a symmetric (not necessarily spherically symmetric) stable process on $R^n (n \geqq 3)$. Then it is known that there exists a potential kernel $G(x, y) = g(x-y)$ (we call it Green function of X) such that

$$\int_0^\infty T_t f(x) dt = \int_{R^n} G(x, y) f(y) dy, \qquad G(x, y) > 0,$$

for each continuous function f of compact support, where $\{T_t\}$ is the semi-group of transition operators for X.

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1) It means that for any $x \in E$ there exists a neighborhood U and exist positive constants C_i, $i=1, 2$ such that $C_1 G_1(X, y) \leqq G_2(x, y) \leqq C_2 G_1(x, y)$, $(x, y) \in U \times U$.

2) This means that each X_1-finely open set is a X_2-finely open set.

3) In one dimensional case, it is easily shown that this phenomenon occurs for unsymmetric stable processes of index α, $0 < \alpha < 1$. Our aim of this note is to give examples of symmetric infinitely divisible processes in higher dimensional spaces.