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*A Two Point Connection Problem for General Linear Ordinary Differential Equations**

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§ 1. Introduction.

In the theory of ordinary differential equations in the complex domain a study of analyzing global behaviours of solutions is of great importance, but extremely difficult. Specifically, such a global problem for linear ordinary differential equations consists in finding explicit connection formulas between local solutions. In fact, as is well known, a fundamental set of solutions of linear differential equations can be expressed in terms of linear combinations of another fundamental set of solutions with constant coefficients. But there exists no general way to evaluate the constant coefficients explicitly.

In 1858, B. Riemann [21] first investigated the connection problem for the so-called hypergeometric differential equation with three regular singularities and derived the complete results by the method of double-circuit contour integration. After that, many authors tackled and to some extent contributed to the global analysis of Fuchsian differential equations, though satisfactory results even for Heun's equation, a second order linear differential equation with four regular singularities have not yet been obtained. For topics on Heun's equation and Fuchsian differential equations, see [7], [5] and [17].

At the quite same time when B. Riemann wrote the above paper, G. G. Stokes [22] had been studying Airy's equation which has only a regular and an irregular singular point in the entire complex plane, and discovered a striking fact that constant coefficients appearing in asymptotic representations of solutions change discontinuously by a change of sectorial neighborhoods of an irregular singular point. This fact is now called the Stokes phenomenon. The Stokes phenomenon, as we explain below, can be completely worked out by the solution of a connection

* A part of the results in this paper was announced in [12] without proofs.