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Principal Oriented Bordism Algebra $\Omega_*(Z_{2^k})$

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Introduction

The principal oriented bordism module $\Omega_*(G)$ for a finite group G is defined to be the module of equivariant bordism classes of closed oriented principal G-manifolds, and is a module over the oriented bordism ring Ω_* of R. Thom (cf. [1]).

This module $\Omega_*(G)$ and the unoriented one $\mathfrak{N}_*(G)$ are studied by several authors. If G is a finite cyclic group, the Ω_*-module structure of $\Omega_*(G)$ is determined by P. E. Conner and E. E. Floyd [1, Ch. VII] for $G=Z_{p^k}$ (p: odd prime), and by K. Shibata [3, §§1–4] for $G=Z_2$. Also it is proved by N. Hassani [2] that there is an isomorphism $\Omega_*(Z_{qr})\cong\Omega_*(Z_q)\otimes_{\Omega_*}\Omega_*(Z_r)$ of Ω_*-modules if q and r are relatively prime.

The main purpose of this note is to study the Ω_*-module structure of $\Omega_*(Z_{2^k})$ for $k>1$. Also, we study the Pontrjagin products in $\Omega_*(Z_{2^k})$ and $\mathfrak{N}_*(Z_{2^k})$ for $k>1$.

In §1, we are concerned with the unoriented bordism module

$$\mathfrak{N}_*(Z_{2^k})\cong\mathfrak{N}_*\otimes H_*(Z_{2^k};\,Z_2)\qquad\text{(cf. [1, (19.3)])}.$$

It is easy to see that this is a free $\mathfrak{N}_*$-module with basis $\{[T,S^{2n+1}],\,i[a,S^{2n}]\,|\,n\geqq 0\}$ (Proposition 1.7), where (T,S^{2n+1}) is the Z_{2^k}-manifold with the diagonal action T of $\exp(\pi\sqrt{-1}/2^{k-1})$ and $i(a,S^{2n})$ is the extension of the Z_2-manifold (a,S^{2n}) with the antipodal action a. Also we study in Theorem 1.22 the product formulae in $\mathfrak{N}_*(Z_{2^k})$ using the results for $\mathfrak{N}_*(Z_2)$ of F. Uchida [6].

In §2, we are concerned with

$$\tilde{\Omega}_n(Z_{2^k})\cong\textstyle\sum_{p+q=n}\tilde{H}_p(Z_{2^k};\,\Omega_q)\qquad\text{(cf. [1, Th. 14.2])}.$$

Using the homomorphism $r\colon\Omega_*(Z_{2^k})\to\mathfrak{N}_*(Z_{2^k})$ obtained by ignoring orientations, and the results for $\Omega_*(Z_2)$ in [3], we prove in Theorem 2.18 that the Ω_*-module $\tilde{\Omega}_*(Z_{2^k})$ $(k>1)$ is a quotient module of the free Ω_*-module

$$\Omega_*\{\{[T,S^{2n+1}],\,iE^{2n+1}W(\omega)\,|\,n\geqq 0,\ \omega\in\pi\}\},$$

where $E^{2n+1}W(\omega)\in\tilde{\Omega}_*(Z_2)$. Finally, we study in Theorem 2.22 the Pontrjagin product in $\tilde{\Omega}_*(Z_{2^k})$.