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Dirichlet Integrals of Functions on a Self-adjoint Harmonic Space

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Introduction

In the previous papers [9], the author introduced a notion of energy for functions on a self-adjoint harmonic space. Our model there was the harmonic space formed by solutions of the self-adjoint second order partial differential equation $\Delta u = Pu$ with $P \geqq 0$ on a Euclidean domain Ω. The energy of a function f with respect to this harmonic space is given by

$$E[f] = D[f] + \int_{\Omega} f^2 P dx, \tag{1}$$

where $D[f]$ denotes the ordinary Dirichlet integral of f over Ω.

For an abstract harmonic space $(\Omega, \mathfrak{H})$, its self-adjointness was defined as the property that it admits a symmetric Green function $G(x, y)$, provided that there is a positive potential on Ω. The condition $P \geqq 0$ in the above model was interpreted as the condition that the constant function 1 is superharmonic. On a self-adjoint harmonic space satisfying this condition, we defined the notion of energy of a function f in terms of potential representation of f with respect to the kernel $G(x, y)$, in such a way that it coincides with $E[f]$ in the special case of the above model.

The definition of energy in [9] also suggests how a value corresponding to the Dirichlet integral $D[f]$ should be defined on such a harmonic space; but it is not clear whether the value has such good properties as the ordinary Dirichlet integral enjoys — among others, whether it is always non-negative.

On the other hand, solutions of the equation $\Delta u = Pu$ form a harmonic space even if P is not necessarily non-negative on Ω (cf., e.g., [7, Théorème 34.1] and [8, Theorem 2.1]), so that one might ask if the method developed in [9] is applicable to the harmonic space on which 1 is not superharmonic. For such a harmonic space, there may not exist positive potentials even if the boundary is large, so that one had better consider the self-adjointness locally. However, in order to make a consistent definition of Dirichlet integrals, some global consideration is also necessary (see § 1.2).

For a self-adjoint harmonic space thus defined, we shal define (in § 4) the notion of *gradient measures* of certain locally bounded functions with the same