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Note on Equivariant Maps from Spheres to Stiefel Manifolds

Toshio Yoshida

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§ 1. Introduction

Let $X=(T, X)$ be a Hausdorff space with a fixed point free involution T. By [2, Def. (3.1)], the *index* of (T, X) is the largest integer n for which there is an equivariant map of the n-sphere S^n into X. The *co-index* of (T, X) is the least integer n for which there is an equivariant map of X into S^n. Here the fixed point free involution of S^n is the antipodal involution A. We abbreviate *index* and *co-index* by $\mathrm{ind}(T, X)$ and $\text{co-ind}(T, X)$, respectively. It may happen for a particular X that there is no upper bound on the dimension of the sphere which can be equivariantly mapped into X; then we write $\mathrm{ind}(T, X)=\infty$. Also if X cannot be equivariantly mapped into S^n no matter how large n, write $\text{co-ind}(T, X)=\infty$.

As there is no equivariant map of S^{n+1} into S^n, we have

$$\mathrm{ind}(A, S^n) = \text{co-ind}(A, S^n) = n.$$

Let $V_{n,m}$ be the Stiefel manifold of orthonormal m-frames in real n-space R^n. There is a fixed point free involution T_2 on $V_{n,m}$ defined by sending an m-frame $(v_1,\dots, v_m)$ to $(-v_1,\dots, -v_m)$.

Let ξ_k be the canonical line bundle over k-dimensional real projective space RP^k, and $n\xi_k$ the Whitney sum of n-copies of ξ_k. Let Span α denote the maximum number of the linearly independent cross-sections of a vector bundle α.

Proposition 1. *$\mathrm{ind}(T_2, V_{n,m})\geqq k$ if and only if $\mathrm{Span}\, n\xi_k\geqq m$.*

For example, $\mathrm{Span}\, n\xi_k$ is studied in [6] and [9].

Corollary 2. *$\mathrm{ind}(T_2, V_{n,2})=\text{co-ind}(T_2, V_{n,2})=n-1$, for even n.*

Remark. By [2, p. 426],

$$n-2 = \mathrm{ind}(T_2, V_{n,2}) < \text{co-ind}(T_2, V_{n,2}) = n-1, \quad \text{for odd} \quad n.$$

Let $Z_q=\{e^{i\theta}|\theta=2\pi h/q, h=0,\dots, q-1\}$ be the cyclic group of order q. Then an action of Z_q on the complex n-space C^n is defined by $e^{i\theta}(z_1,\dots, z_n)=(e^{i\theta}z_1,\dots, e^{i\theta}z_n)$.