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A Comparison Theorem on Generalized Capacity

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1. Introduction.

For any real function $\phi(r)$ which is continuous and monotone decreasing for $r>0$ with $\lim_{r\to 0+}\phi(r)=+\infty$, Frostman [2] defined capacity C^{ϕ} with respect to ϕ. Let ϕ_0 be a fixed function of the same type and let us consider the following two properties.

i) *If* $C^{\phi_0}(K)=0$ *for some compact set* $K\subset R^d$, *then* $C^{\phi}(K)=0$ *and the converse implication is also valid.*

ii) $M_1\phi_0(r)\geqq\phi(r)\geqq M_2\phi_0(r)$ *for each* $0<r<\delta_0$, *where* M_i, $i=1, 2$ *are positive constants.*

It is evident by the definition of capacity that ii) implies i). If ϕ_0 is such that $r^d\phi_0(r)$ is monotone increasing with $\lim_{r\to 0+} r^d\phi_0(r)=0$ and $r^{-d}\int_0^r \phi_0(s)s^{d-1}\,ds\leqq M_3\phi_0(r)$ for $0<r<\delta$, we see that i) implies $\phi(r)\leqq M_4\phi_0(r)$ by Theorem 4 and Remark in S. J. Taylor [6]. Our object in the present note is to show that i) implies $M_5\phi_0(r)\leqq\phi(r)$ for $0<r<\delta$ in case $r^p\phi_0(r)$ is monotone increasing for some $0<p<d$, which is a stronger assumption on ϕ_0 than S. J. Taylor's. Our result is as follows.

THEOREM. *Let* $\phi_0(r)$ *and* $\phi(r)$ *be such that they are monotone decreasing, right continuous with* $\lim_{r\to 0+}\phi_0(r)=\lim_{r\to 0+}\phi(r)=+\infty$ *and* $r^p\phi_0(r)$ *is monotone increasing for some* $d>p>0$. *Then* i) *implies* ii).

2. Definitions and known results.

We set

$\Phi=\{\phi;\ \phi(r)$ *is positive, monotone decreasing and right continuous with* $\lim_{r\to 0+}\phi(r)=+\infty\}$,

and

$$\phi_p=\{\phi\in\Phi;\ r^p\phi(r) \textit{ is monotone increasing for } 0<r<\delta\}$$

For a compact set K in Euclidean d-space R^d we set

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