HIROSHIMA MATH. J.
5 (1975), 61–67

A Note on $G(\mathfrak{a})$-Domains and Hilbert Rings

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(Received September 4, 1974)

In a recent paper [1], we defined the property $J(A)$ for an integral domain R, which is useful to prove a generalized Hilbert Nullstellensatz. At that time, we restricted ourselves to prime ideals of height one. However, we can readily see that Lemma 1, Lemma 2 and Proposition 1 in Section 1 of [1] are valid, if we replace the set $Ht_1(R)$ of prime ideals of height one (resp. $H_R(D)$) by the set $P(R)$ of non zero prime ideals (resp. $H_R^*(D)$ (see the definition below)). So, in this paper, we define the property $J^*(\mathfrak{a})$ for a cardinal number $\mathfrak{a}$ in place of the property $J(\mathfrak{a})$; here the cardinal number $\mathfrak{a}$ will always be assumed not less than $\aleph_0$, because if $\mathfrak{a}$ is finite, then it is clear that an integral domain R has the property $J^*(\mathfrak{a})$ if and only if R is not a G-domain (see the definition in [4]). Also, by taking account of the fact mentioned above, we define $G(\mathfrak{a})$-domain as a concept against the property $J^*(\mathfrak{a})$, and furthermore by introducing the notion of $G(\mathfrak{a})$-ideal and $H(\mathfrak{a})$-ring similar to G-ideal and Hilbert ring in [4], we can obtain some results generalizing those in [3] and [4].

The author wishes to express his thanks to Professor M. Nishi for his valuable advice and his comments in writing this paper.

1. $G(\mathfrak{a})$-domains

All rings considered are commutative with identity. Let $\mathfrak{a}$ be a cardinal number not less than $\aleph_0$. We say that a polynomial ring over R is an $\mathfrak{a}$-polynomial ring over R if the cardinality of the set of its variables is $\mathfrak{a}$, and we say that an R-algebra A is $\mathfrak{a}$-generated over R if A is an R-homomorphic image of the $\mathfrak{a}$-polynomial ring over R. Call a subset D of an integral domain R a $J(\mathfrak{a})$-subset if D does not contain zero element and if the cardinality of D is not greater than $\mathfrak{a}$. A bit of notation: For an integral domain R, we denote by $P(R)$ the set of non zero prime ideals in R, $Ht_1(R)$ the set of prime ideals of height one, and for a given subset E of R we denote by $H_R^*(E)$ the set of non zero prime ideals in R which contains at least one element of E, $H_R(E)$ the set of prime ideals of height one in R which contains at least one element of E.

DEFINITION. Let R be an integral domain. When $H_R(D)$ is properly contained in $Ht_1(R)$ for any $J(\mathfrak{a})$-subset D of R, then we say that the ring R has the property $J(\mathfrak{a})$. When $H_R^*(D)$ is properly contained in $P(R)$ for any $J(\mathfrak{a})$-subset D of R, then we say that the ring R has the property $J^*(\mathfrak{a})$.