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Oscillations of Differential Inequalities with Retarded Arguments

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In this paper we consider the following differential inequalities with retarded arguments:

$$\text{(A)} \qquad (-1)^n x^{(n)}(t) \operatorname{sgn} x(t) \geqq \sum_{i=1}^{N} p_i(t) f_i(x(g_i(t))),$$

$$\text{(B)} \qquad (-1)^n x^{(n)}(t) \operatorname{sgn} x(t) \geqq p_0(t) \prod_{i=1}^{N} \phi_i(x(g_i(t))).$$

For these inequalities the following conditions will be assumed without further mention:

(a) The functions $p_i(t)$ $(i=0, 1,\dots, N)$ are continuous and nonnegative on $[0, \infty)$.

(b) The functions $f_i(y)$ and $\phi_i(y)$ $(i=1,\dots, N)$ are continuous and positive on $(-\infty, 0) \cup (0, \infty)$ and $f_i(y) \operatorname{sgn} y$ and $\phi_i(y) \operatorname{sgn} y$ are nondecreasing in y.

(c) The functions $g_i(t)$ $(i=1,\dots, N)$ are continuous and nondecreasing on $[0, \infty)$ and

$$g_i(t) \leqq t \quad \text{for} \quad t \geqq 0 \quad \text{and} \quad \lim_{t\to\infty} g_i(t) = \infty.$$

We shall restrict our attention to solutions $x(t)$ of (A) or (B) which exist on a half-line $[t_x, \infty)$. Such a solution is called oscillatory if it has a sequence of zeros tending to infinity; otherwise a solution is called nonoscillatory.

The object of this paper is to obtain sufficient conditions under which all bounded solutions of the differential inequalities (A) and (B) are oscillatory. Our results generalize the results due to Gustafson [1] and Shreve [8]. For related results we refer the reader to the papers by Koplatadze [2], Kusano and Onose [3], Ladas [4], Ladas, Lakshmikantham and Papadakis [5], and Sficas and Staikos [6, 7].

THEOREM 1. *Assume that*

$$\text{(1)} \qquad \limsup_{t\to\infty} \int_{g(t)}^{t} [s-g(t)]^{n-1} \sum_{i=1}^{N} p_i(s) ds > (n-1)! \limsup_{y\to 0} \frac{|y|}{f(y)},$$

where $f(y) = \min_{1\leqq i\leqq N} f_i(y)$ *and* $g(t) = \max_{1\leqq i\leqq N} g_i(t)$.