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Periodic Solutions for Certain Time-dependent Parabolic Variational Inequalities

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Introduction

For a real Banach space V we denote by V^* the dual space of V, by $\|\cdot\|_V$ and $\|\cdot\|_{V^*}$ the norms in V and V^*, respectively, and by $(\cdot,\cdot)_V$ the natural pairing between V^* and V. A (multivalued) operator A from a Banach space V into its dual V^* (i.e., assigning to each $v \in V$ a subset Av of V^*) is called monotone if

$$(v^* - w^*, v - w)_V \geqq 0 \qquad \text{for any} \quad [v, v^*], [w, w^*] \in G(A),$$

where $G(A)$ is the graph of the operator A, i.e.,

$$G(A) = \{[v, v^*] \in V \times V^* \colon v \in D(A) \text{ and } v^* \in Av\}$$

with $D(A) = \{v \in V\colon Av \neq \phi\}$. If A is monotone and there is no proper monotone extension of A, then A is called maximal monotone.

Throughout this paper we let H be a Hilbert space and X a Banach space such that $X \subset H$, X is dense in H and the natural injection from X into H is continuous, and suppose that X is uniformly convex and X^* is strictly convex. Identifying H with its dual space by means of the inner product $(\cdot,\cdot)_H$ in H, we have the relation $X \subset H \subset X^*$. By the symbols "$\xrightarrow{s}$" and "$\xrightarrow{w}$" we mean the convergence in the strong and weak topology, respectively.

Let $0 < T < \infty$, $2 \leqq p < \infty$ and $1/p + 1/p' = 1$ and let ψ be an extended real-valued function on $[0, T] \times X$ such that for each $t \in [0, T]$, $\psi(t; \cdot)$ is a lower semicontinuous convex function on X with values in $(-\infty, +\infty]$, $\psi(t; \cdot) \not\equiv +\infty$, and such that for each $v \in L^p(0, T; X)$, $t \to \psi(t; v(t))$ is measurable on $[0, T]$. We define a functional Ψ on $L^p(0, T; X)$ by

$$\Psi(v) = \begin{cases} \displaystyle\int_0^T \psi(t; v(t))dt & \text{if} \quad v \in D(\Psi), \\ +\infty & \text{otherwise}, \end{cases}$$

where $D(\Psi) = \{v \in L^p(0, T; X)\colon t \to \psi(t; v(t)) \text{ is integrable on } (0, T)\}$.

We now pose the following problem: Given an $f \in L^{p'}(0, T; X^*)$, find a $u \in D(\Psi) \cap C([0, T]; H)$ such that

(i) $u(0) = u(T)$,