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Codivisorial and Divisorial Modules over Completely Integrally Closed Domains (II)

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Introduction

In our paper [5], we have introduced an operation on modules over a completely integrally closed domain, which we called "*divisorial envelope*", and we have studied some basic properties of the divisorial envelope of a codivisorial module and also developed a theory of codivisorial and divisorial modules which shows us that the intrinsic nature of codivisorial and divisorial modules over a Krull domain is similar to that of modules over a Dedekind domain.

The fundamental theorem of finitely generated abelian groups is based on the fact that the ring of rational integers is a principal ideal domain, in other words, a ring in which every ideal is free. It is well known that the above theorem is generalized to finitely generated modules over a Dedekind domain which is characterized by the property that any ideal is projective. It seems plausibe to the authors that the theorem can be formulated for modules over a Krull domain as far as we are concerned with codivisorial and divisorial modules. In fact, in [3], N. Bourbaki dealt with the case of noetherian Krull domains. The main purpose of this Part II is to introduce the notion of an essentially finite module over a Krull domain and develop a theory of invariants by making use of the divisorial envelope.

§3. Divisorial equivalence

Throughout this §, A is always a strongly integrally closed domain, unless otherwise specified.

PROPOSITION 30. *Let $f\colon M\to N$ be a homomorphism of A-domules and $p\colon M\to M/\tilde{M}$, $q\colon N\to N/\tilde{N}$ be the canonical projections.*

(i) *There is a unique homomorphism $f_*\colon M/\tilde{M}\to N/\tilde{N}$ such that $f_*p=qf$.*

(ii) *If f is pseudo-injective, then f_* is injective, and if f is pseudo-isomorphic, then so is f_*.*

(iii) *If f is pseudo-isomorphic and M is divisorial, then f_* is an isomorphism.*

PROOF. The existence of f_* follows from Prop. 3 and the uniqueness is