HIROSHIMA MATH. J.
5 (1975), 439–446

A Note on Subloops of a Homogeneous Lie Loop and Subsystems of its Lie Triple Algebra

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(Received May 10, 1975)

Introduction

In our previous paper [3], we have introduced a concept of a geodesic homogeneous Lie loop G which is a generalization of the concept of Lie groups, and shown that the tangent space $\mathfrak{G}$ at the identity of G forms a Lie triple algebra under the operations defined by the torsion and curvature tensors of the canonical connection of G, and that $\mathfrak{G}$ characterizes locally the homogeneous Lie loop G (cf. [3, Definitions 3.1, 3.5 and Theorems 7.2, 7.3, 7.8]).

In this paper, we observe the correspondence between the set of Lie subloops of G and the set of subsystems[1)] of $\mathfrak{G}$, and show the following main theorem:

THEOREM 1. *Let G be a connected geodesic homogeneous Lie loop and $\mathfrak{G}$ its Lie triple algebra. Then, for any connected left invariant Lie subloop H of G, the Lie triple algebra $\mathfrak{H}$ of H is a left invariant subsystem of $\mathfrak{G}$.*

Conversely, for any left invariant subsystem $\mathfrak{H}$ of $\mathfrak{G}$, there exists a unique connected left invariant Lie subloop H of G whose Lie triple algebra is $\mathfrak{H}$.

Here, we call a subloop H of G (resp. subsystem $\mathfrak{H}$ of $\mathfrak{G}$) *left invariant* if it is invariant under the left inner mapping group $L_0(G)$ of G (resp. the group $dL_0(G)$ of linear transformations of $\mathfrak{G}$ induced from $L_0(G)$).

It should be noted that, when G is reduced to a Lie group, the above theorem is reduced to the well known theorem of the correspondence of Lie subgroups of G and Lie subalgebras of the Lie algebra $\mathfrak{G}$ of G.

The notations and terminologies used in this paper are all refered to [3].

§1. Local subloops of a homogeneous Lie loop

To study local subloops of a geodesic local Lie loop in a locally reductive space, we consider its auto-parallel submanifolds. Let M be a differentiable manifold with a linear connection ∇. A submanifold N of M is called *autoparallel* if, for each vector X tangent to N at any x and for each piecewise differentiable

1) By a *subsystem* of a Lie triple algebra $\mathfrak{G}$, we mean a subalgebra of $\mathfrak{G}$ which is closed under the ternary operation of $\mathfrak{G}$.