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Nonoscillation Generating Delay Terms in Even Order Differential Equations

R. S. DAHIYA

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1. Introduction

The study of differential equations with time lag is growing increasingly significant due to technological dependence on physical systems with after effects. Mathematically, such systems [1] are governed by some sort of differential equation with an appropriate delay term which in itself may be a variable quantity. The oscillatory behavior of such equations becomes an interesting phenomenon especially when delay is chiefly responsible for causing oscillations. For example, following Teodorick [20], (also see Norkin [12, pp. 4-6]), the equation

$$x''(t)+\frac{r}{m}x'(t)+\frac{k}{m}x(t)+\frac{2p}{\pi am}x(t-\Delta)=0 \tag{1}$$

represents the working of an electric hammer of mass m. A study of this system shows that without the delay term Δ, there will be no vibrations.

Results concerning the oscillatory behavior of a wide variety of retarded equations can be found in [2, 3, 5, 9, 10, 13, 16, 19, 21]. However most of these results are such that the delay term does not play any role at all. But an obvious example such as

$$y''(t)-y(t-\pi)=0 \tag{2}$$

clearly indicates by its solutions $\sin t$ and $\cos t$, that its oscillatory behavior is different from that of the ordinary differential equation

$$y''(t)-y(t)=0 \tag{3}$$

which is nonoscillatory. This difference in the behavior of equations (2) and (3) is clearly due to the delay term π.

Recently Ladas and Lakshmikantham [10] showed that if $p(t)>0$, $p'(t)\leq 0$ and $\tau^2 p(t)\geq 2$, then the bounded solutions of the equation

$$y''(t)-p(t)y(t-\tau)=0 \tag{4}$$

are oscillatory. Taking $p(t)\equiv 1$ and $\tau=0$, equation (4) reduces to equation (3) which we know is nonoscillatory. Ladas, Ladde and Pappadakis in [9, Theorem