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Oscillation and a Class of Odd Order Linear Differential Equations

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Introduction

Let q be a continuous function from $[0, \infty)$ to $(0, \infty)$. In studying oscillation for

$$u^{(m)}+qu = 0, \tag{1}$$

and related equations, many authors have recognized that the even and odd order cases have some fundamental differences. See, for example, A. G. Kartsatos [6], T. Kusano and H. Onose [10], G. Ladas, V. Lakshmikanthan, and J. S. Papadakis [11], Y. G. Sficas [15], Sficas and V. A. Staikos [16], and G. H. Ryder and D. V. V. Wend [14]. On the other hand, Ladas, Lakshmikantham, and Papadakis [11], Sficas and Staikos [16], and the present author [12] have observed that, for some purposes, the odd and even order cases coalesce if one replaces (1) by

$$u^{(m)}+(-1)^m qu = 0. \tag{2}$$

For example, the present author [12] has shown that

$$\int_0^\infty t^{m-1}q(t)dt < \infty$$

is a necessary and sufficient condition for the existence of a bounded nonoscillatory solution of (2), irrespective of the parity of m.

In the even order case, it is known that

$$\int_0^\infty t^{2n-2}q(t)dt = \infty \tag{3}$$

implies that every solution of

$$u^{(2n)}+qu = 0 \tag{4}$$

is oscillatory (see, for example, G. V. Anan'eva and V. I. Balaganskii [2], H. C. Howard [4], I. T. Kiguradze [8], V. A. Kondrat'ev [9], and C. A. Swanson [17, p. 175]). If (3) fails, the present author [13] has found two continuous