HIROSHIMA MATH. J.
6 (1976), 183–189

Remarks on the Oscillatory Behavior of Solutions of Functional Differential Equations with Deviating Argument

Takaŝi KUSANO and Hiroshi ONOSE

(Received September 12, 1975)

1. Introduction

We consider the following nth order functional differential equations with deviating argument

$$(r_{n-1}(t)(r_{n-2}(t)(\cdots(r_2(t)(r_1(t)y'(t))')'\cdots)')')' + (-1)^n y(g(t))F([y(g(t))]^2, t) = 0, \tag{1}$$

$$(r_{n-1}(t)(r_{n-2}(t)(\cdots(r_2(t)(r_1(t)y'(t))')'\cdots)')')' + (-1)^{n+1} y(g(t))F([y(g(t))]^2, t) = 0, \tag{2}$$

$$(r_{n-1}(t)(r_{n-2}(t)(\cdots(r_2(t)(r_1(t)y'(t))')')\cdots')')' + y(g(t))F([y(g(t))]^2, t) = 0. \tag{3}$$

The conditions we always assume for r_i, g, F are as follows:

(a) $g(t)$ is continuous on $[\tau, \infty)$ and $\lim_{t\to\infty} g(t) = \infty$;

(4) (b) each $r_i(t)$ is continuous and positive on $[\tau, \infty)$, and

$$\int_\tau^\infty \frac{dt}{r_i(t)} = \infty, \qquad i = 1,\dots, n-1 ;$$

(c) $F(z, t)$ is nonnegative on $(0, \infty) \times [\tau, \infty)$. $yF(y^2, t)$ is continuous on $(-\infty, \infty) \times [\tau, \infty)$ and is nondecreasing in y for each $t \geqq \tau$.

We restrict our discussion to those solutions $y(t)$ of the above differential equations which exist on some ray $[T_y, \infty)$ and satisfy

$$\sup\{|y(t)|: t_0 \leqq t < \infty\} > 0$$

for every $t_0 \in [T_y, \infty)$. Such a solution is said to be oscillatory (or to oscillate) if it has arbitrarily large zeros. Otherwise the solution is said to be nonoscillatory.