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Principal Oriented Bordism Modules of Generalized Quaternion Groups

Yutaka KATSUBE

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Introduction

The principal oriented bordism module $\Omega_*(G)$ of a group G is defined to be the module of all equivariant bordism classes of closed principal oriented (smooth) G-manifolds. $\Omega_*(G)$ is a module over the oriented bordism ring Ω_*, and this module $\Omega_*(G)$ and the unoriented one $\mathfrak{N}_*(G)$ are studied by several authors.

The purpose of this paper is to determine the Ω_*-module structure of $\Omega_*(H_m)$, $m \geqq 2$, where H_m is the generalized quaternion group generated by two elements x and y with two relations

$$x^{2^{m-1}} = y^2 \quad \text{and} \quad xyx = y,$$

that is, the subgroup of the unit sphere S^3 in the quaternion field $\boldsymbol{H}$ generated by $x = \exp(\pi i/2^{m-1})$ and $y = j$.

The group H_m acts freely on the unit sphere S^{4n+3} in the quaternion $(n+1)$-space $\boldsymbol{H}^{n+1}$ by the diagonal action $\alpha_m(q, (q_0, \ldots, q_n)) = (qq_0, \ldots, qq_n)$ $(q, q_i \in \boldsymbol{H})$, and we obtain the principal oriented H_m-manifold

$$(0.1) \qquad (\alpha_m, S^{4n+3}) \qquad (n \geqq 0).$$

Also, the element $x = \exp(\pi i/2^{m-1})$ generates the cyclic subgroup Z_{2^m} of order 2^m, and this group acts on the unit sphere S^{2n+1} in the complex $(n+1)$-space $\boldsymbol{C}^{n+1}$ by the diagonal action $x(z_0, \ldots, z_n) = (xz_0, \ldots, xz_n)$ $(z_i \in \boldsymbol{C})$. We denote this Z_{2^m}-manifold by (T_m, S^{2n+1}). Hence we obtain the extension

$$(0.2) \qquad i_m(T_m, S^{4n+1}) \qquad (n \geqq 0),$$

by the inclusion $i_m\colon Z_{2^m} \subset H_m$, which is the disjoint union $Z_2 \times S^{4n+1}$ with the H_m-action given by

$$x(\varepsilon, z) = (\varepsilon, x^{\varepsilon} z), \quad y(\varepsilon, z) = (-\varepsilon, \varepsilon z) \qquad (\varepsilon = \pm 1, z \in S^{4n+1}).$$

Let π be the set of partitions $\omega = (a_1, \ldots, a_r)$ with unequal parts a_j, none of which is a power of 2. By the consideration of K. Kawakubo [6], there is a Z_2-manifold