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Generalized Extremal Length of an Infinite Network

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Introduction

The extremal length of a network, which is the reciprocal of the value of a quadratic programming problem, was first investigated by R. J. Duffin [4] on a finite graph and next by the second author [7] on an infinite graph. In this paper we shall be concerned with a generalized form of the extremal length as in [5] along the same lines as in [4] and [7]. The generalized extremal length of an infinite network may be regarded as the reciprocal of the value of a convex programming problem. One of our main purposes is to establish a reciprocal relation between the generalized extremal distance and the generalized extremal width of an infinite network which was established by M. Ohtsuka [5] for the continuous case. We shall also study the generalized extremal length of an infinite network relative to a finite set and the ideal boundary of the network. A concept of non-linear flows which was studied in [1] and [3] will appear in § 3 and § 4 in connection with the extremal width of a network.

§ 1. Preliminaries

Let X be a set of nodes and let Y be a set of directed arcs. Since we always consider the case where X and Y consist of a countably infinite number of elements, we put

$$X = \{0, 1, 2,\dots, n,\dots\},$$

$$Y = \{1, 2,\dots, n,\dots\}.$$

Let $K=(K_{vj})$ be the node-arc incidence matrix. Namely $K_{vj}=1$ if arc j is directed toward node v, $K_{vj}=-1$ if arc j is directed away from node v and $K_{vj}=0$ if arc j and node v do not meet.

We assume that X, Y and K satisfy the following conditions:

(1.1) $\{j \in Y;\ K_{vj} \neq 0\}$ is a nonempty finite set for each $v \in X$.

(1.2) $e(j)=\{v \in X;\ K_{vj} \neq 0\}$ consists of exactly two nodes for each $j \in Y$.

(1.3) For any $\alpha, \beta \in X$, there are $v_1,\dots, v_n \in X$ and $j_1,\dots, j_{n+1} \in Y$ such that $e(j_i)=\{v_{i-1}, v_i\}$, $i=1,\dots, n+1$ with $v_0=\alpha$ and $v_{n+1}=\beta$.