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Forced Oscillations in General Ordinary Differential Equations with Deviating Arguments

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1. Introduction

Recently quite a few authors have spent considerable effort in finding conditions to ensure that nonoscillatory solutions of both ordinary and their companion retarded differential equations approach zero asymptotically. For these criteria, the reader is referred to [3, 5, 6, 8, 9] and references cited in them. However the literature is very scanty about similar results in regard to oscillatory solutions of these equations. Our purpose here is to find conditions to ensure that the oscillatory solutions of the general n-th order equation

$$(1) \qquad (r(t)y'(t))^{(n-1)} + a(t)y_\tau(t) = f(t), \; y_\tau(t) \equiv y(t-\tau(t))$$

approach to zero as $t \to \infty$.

We now give definitions and assumptions that hold in the rest of this paper:

(i) $\tau(t)$, $r(t)$, $a(t)$, $f(t)$ are real, continuous and defined on the whole real line R.

(ii) $r(t)$ and $\tau(t)$ are positive on R. $\tau(t)$ is bounded above by $K_0 > 0$.

We call a function $h(t) \in C[0, \infty)$ oscillatory if it has arbitrarily large zeros. Otherwise $h(t)$ is called nonoscillatory on the half line $[0, \infty)$.

In what follows only continuous and extendable solutions of equations (1) and (2) will be considered. The term "solution" applies only to such solutions in this manuscript.

2. Main results

LEMMA 1. *Suppose $p_1 > p_2 > p_3 > p_4 > \cdots > p_{n-2}$ are respectively the zeros of*

$$(r(t)y'(t))', \; (r(t)y'(t))'', \ldots, \; (r(t)y'(t))^{(n-3)}, \; (r(t)y'(t))^{(n-2)},$$

where $y(t)$ is a solution of equation (1). *Further suppose that $t_1 < p_{n-2}$ and $t_2 > p_1$ are zeros of $y(t)$. Suppose*