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Remarks on the Asymptotic Relationships between Solutions of Two Systems of Ordinary Differential Equations

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1. Introduction

Consider the system of ordinary differential equations

$$x' = A(t)x + f(t, x), \qquad t \geqq t_0, \tag{1}$$

where x is an n-vector, $A(t)$ is a continuous $n \times n$ matrix function on $I = [t_0, \infty)$, and $f(t, x)$ is a continuous n-vector function of t and x on $I \times R^n$. Recently, Rab [7] has taken up the case where all components f_j of f depend only on t and some of the components of x, say, $x_{i_1},\dots, x_{i_q}$, $1 \leqq i_1 < \cdots < i_q \leqq n$, and has presented conditions which lead to an equivalence between certain components of the solutions of the system (1) and certain components of the solutions of the unperturbed system

$$y' = A(t)y, \qquad t \geqq t_0. \tag{2}$$

He has shown in particular that the first theorem of Hallam [6] concerning the second order scalar differential equations

$$x'' = a(t)x + f(t, x), \qquad y'' = a(t)y \tag{3}$$

follows from his theorem as a corollary.

The purpose of this note is to establish a theorem which improves considerably the above mentioned results of Rab and to provide some examples demonstrating its application to specific classes of differential equations. In particular it is shown that our result, when applied to (3), yields the second theorem of Hallam [6] which is not covered by Ráb's result.

2. Main result

We assume that the components f_j of f depend essentially on t and the q components $x_{i_1},\dots, x_{i_q}$ $(1 \leqq i_1 < \cdots < i_q \leqq n)$ of x in the sense that

$$|f_j(t, x_1,\dots, x_n)| \leqq \omega_j(t, |x_{i_1}|,\dots, |x_{i_q}|) \tag{4}$$

for $(t, x) \in I \times R^n$ and $j = 1,\dots, n$, where each $\omega_j(t, r_1,\dots, r_q)$ is continuous on I