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Simplexes and Dirichlet Problems on Locally Compact Spaces

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Introduction

Let Ω be a bounded open set in a euclidean space and f a continuous function defined on the boundary $d\Omega$. The classical Dirichlet problem asks for a continuous function u on the closure $\overline{\Omega}$ of Ω which is harmonic in Ω and equal to f on $d\Omega$. H. Bauer [1] considered an analogous abstract Dirichlet problem for a compact Hausdorff space X and a vector space B of real-valued, continuous functions on X which contains constant functions and separates points of X. He investigated conditions with which a continuous function f defined on the closure of the Choquet boundary $\delta(E)$ with respect to B can be extended to X as a function of B or a **B**-affine function. In the special case where X is a convex compact set in a locally convex real vector space and B is the vector space of the restrictions to X of all functions of the form $f+\alpha$ with a linear functional f and a constant function α, Bauer proved that $\mathscr{C}(\overline{\delta(\mathbf{B})})=\mathbf{B}|\overline{\delta(\mathbf{B})}$ if and only if B is a simplex and $\delta(\mathbf{B})$ is closed ([1, Satz 13]). Thus the abstract Dirichlet problem is deeply connected with the theory of simplexes (see [5] and [6]). Similar abstract Dirichlet problems on a compact set and their relations with the theory of simplexes have been discussed by many authors; e.g., [3] and [8].

In the case where X is a locally compact and σ-compact Hausdorff space, G. Mokobodzki and D. Sibony ([9], [10]) showed that the Choquet boundary with respect to a certain convex cone C of lower semicontinuous functions on X is not empty, using the notion of adapted cones due to G. Choquet [5].

Let P be an adapted convex cone consisting of non-negative continuous functions on X and C be a convex cone consisting of P-bounded continuous functions on X. We shall show that many results in [1], [3], [8] concerning simplexes and abstract Dirichlet problems, which are obtained for a compact space X, are also valid with respect to such a cone C in the case where X is a locally compact and σ-compact space. We shall then apply these results to Dirichlet problems for arbitrary open or closed sets in Bauer's axiomatic potential theory ([2]).

Most of the results in this paper were announced in [15] and [16]. Since the proofs in those papers are sketchy, we shall give details in the present paper.

Here we remark that recently J. Bliedtner and W. Hansen (Inventions math.