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Remarks on the Multiplicative Products of Distributions

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Many attempts have been made for defining the multiplication between distributions. Y. Hirata and H. Ogata [3] have defined a product of distributions and J. Mikusiński [9] has also defined the same product in a different fashion. In [5], we have considered the multiplication invariant under diffeomorphism which covers the multiplication in the above sense. If S, $T\in\mathscr{D}'(R^N)$ and if $\alpha S*\check{T}$ has the value $(\alpha S*\check{T})(0)$ at 0 in the sense of S. Łojasiewicz [8] for any $\alpha\in\mathscr{D}(R^N)$, then there exists a unique distribution W such that $\langle W,\alpha\rangle=(\alpha S*\check{T})(0)$. In [10], R. Shiraishi has defined a restricted δ-sequence $\{\rho_n\}$ as a sequence of non-negative functions $\rho_n\in\mathscr{D}(R^N)$ such that

(i) $\operatorname{supp}\rho_n$ converges to $\{0\}$ as $n\to\infty$

(ii) $\int\rho_n(x)dx$ converges to 1 as $n\to\infty$;

(iii) $\int|x|^{|p|}|D^p\rho_n(x)|dx\leqq M_p$ (M_p being independent of n),

where the integral is extended over the whole N-dimensional space, and he has shown that the existence of the product $W=S\circ T$ of S and T is equivalent to each of the following conditions:

(1) The distributional limit $\lim_{n\to\infty}(S*\rho_n)(T*\tilde{\rho}_n)$ exists for every restricted δ-sequences $\{\rho_n\}$ and $\{\tilde{\rho}_n\}$:

(2) The distributional limit $\lim_{n\to\infty}(S*\rho_n)T$ exists for every restricted δ-sequence $\{\rho_n\}$.

(3) The distributional limit $\lim_{n\to\infty}S(T*\rho_n)$ exists for every restricted δ-sequence $\{\rho_n\}$

And if one of these conditions is satisfied, the limit equals W.

On the other hand, we may define the multiplicative product $S\vartriangle T$ as the distributional limit $\lim_{n\to\infty}(S*\rho_n)(T*\rho_n)$, if it exists for every restricted δ-sequence $\{\rho_n\}$ ([10, p. 97]). The purpose of this paper is to investigate this multiplication Δ by making a comparison with the multiplication $\circ$.

By the definition stated above we see that if $S\circ T$ exists, then $S\vartriangle T$ exists and is equal to $S\circ T$. However the converse does not hold ([10, p. 97]).