Meromorphic Mappings of a Covering Space over C^m into a Projective Variety and Defect Relations

Junjiro NOGUCHI

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1. Introduction

Carlson-Griffiths [1] and Griffiths-King [4] studied the value distribution of holomorphic mappings from a smooth affine variety A into a smooth projective variety V. Among others, they established Nevanlinna's second main theorem and defect relation for holomorphic mappings from A into V. Recently, these results were generalized to the case of meromorphic mappings by Shiffman [13].

In the present paper we study the value distribution of meromorphic mappings from X into V, where X is the complex space of a finite analytic covering $X \xrightarrow{\pi} C^m$ (see Definition 1 in section 2). The main purpose is to show Nevanlinna's second main theorem and defect relation of Griffiths-King's type for meromorphic mappings from X into V (see Theorems 1 and 2 in section 6 and cf. [4]).

The next section will be devoted to the notation and terminologies. In section 3 we shall prove two preparatory lemmas concerning positive currents on X. In section 4 we shall generalize the ramification estimate in Selberg [12] to the case of the finite analytic covering $X \xrightarrow{\pi} C^m$ (Lemma 4.1). This estimate and the use of a singular volume form on V constructed by Carlson-Griffiths [1] and Griffiths-King [4] will play essential roles to obtain the second main theorem in section 6. In section 5 we shall investigate the proper domain of existence of a meromorphic mapping $f: X \rightarrow V$. This investigation will make the ramification estimate, obtained in section 4, possible to apply to the proof of the second main theorem. In the same section we shall prove that the characteristic function T(r, L) of a meromorphic mapping $f: X \rightarrow V$ with respect to a positive line bundle $L \rightarrow V$ (see section 3 for the definition) satisfies

$$T(r, L) = O(\log r)$$

if and only if the finite analytic covering $X \xrightarrow{\pi} C^m$ is algebraic (see Definition 2 in section 2) and f is rational, provided that f separates the fibres of $X \xrightarrow{\pi} C^m$ (see Definition 3 in section 2). This is a fundamental property of the growth of the characteristic function T(r, L).