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On the Oscillation of Solutions of Nonlinear Functional Differential Equations

Takaŝi KUSANO and Hiroshi ONOSE

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1. Introduction

This paper is concerned with nonlinear functional differential equations with deviating arguments of the form

$$\text{(A)} \qquad x^{(n)}(t)+f(t,\ x<\boldsymbol{g}_0(t)>,\ [x']^2<\boldsymbol{g}_1(t)>,\dots,\ [x^{(n-1)}]^2<\boldsymbol{g}_{n-1}(t)>) = 0,$$

where $n \geqq 2$, $\boldsymbol{g}_i(t)=(g_{i1}(t),\dots, g_{im_i}(t))$, $i=0, 1,\dots, n-1$,

$$x<\boldsymbol{g}_0(t)> = (x(g_{01}(t)),\dots, x(g_{0m_0}(t))),$$

and

$$[x^{(i)}]^2<\boldsymbol{g}_i(t)> = ([x^{(i)}(g_{i1}(t))]^2,\dots, [x^{(i)}(g_{im_i}(t))]^2), \qquad i = 1,\dots, n-1.$$

The conditions we always assume for f, g_{ij} are as follows:

(a) $f(t, \boldsymbol{y}_0, \boldsymbol{y}_1,\dots, \boldsymbol{y}_{n-1})$ is continuous on the set $[t_0, \infty)\times E$, where

$$E=R^{m_0}\times R_+^{m_1}\times\cdots\times R_+^{m_{n-1}} \quad (R=(-\infty, \infty),\ R_+=[0, \infty)),$$

$$f(t, \boldsymbol{y}_0, \boldsymbol{y}_1,\dots, \boldsymbol{y}_{n-1})>0 \text{ if } \boldsymbol{y}_0>0, \text{ and}$$

$$f(t, -\boldsymbol{y}_0, \boldsymbol{y}_1,\dots, \boldsymbol{y}_{n-1})=-f(t, \boldsymbol{y}_0, \boldsymbol{y}_1,\dots, \boldsymbol{y}_{n-1}).$$

(b) $g_{ij}(t)$, $j=1,\dots, m_i$, $i=0, 1,\dots, n-1$, are continuous on $[t_0, \infty)$ and $\lim_{t\to\infty} g_{ij}(t)=\infty$.

In what follows we restrict our discussion to those solutions $x(t)$ of equation (A) which exist on some half-line $[T_x, \infty)$ and satisfy

$$\sup\{|x(t)|: t \geqq T\} > 0$$

for every $t\geqq T_x$. Such a solution is called oscillatory if the set of its zeros is not bounded above. Otherwise the solution is called nonoscillatory. A nonoscillatory solution is said to be strongly nomotone if it tends monotonically to zero as $t\to\infty$ together with its first $n-1$ derivatives.

The objective of this paper is to study the oscillatory behavior of solutions of equation (A) with specific nonlinearity defined below. We provide conditions