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Smooth S^3-Actions on n Manifolds for $n \leqq 4$

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§1. Introduction

In this note, we say that M is an S^3 ($=SU(2)$)-manifold, if M is a connected compact smooth manifold admitting a non-trivial smooth S^3-action $S^3 \times M \to M$. The purpose of this note is to classify such closed manifolds of dimension less than 5 by S^3-equivariant diffeomorphisms.

We notice the following results (cf. [1, Cor. 3.2] and [6, Th. 2.6.7]).

(1.1) Any closed proper subgroup of

$$S^3 = \{q \in H;\ |q| = 1\} \qquad (H \text{ is the quaternion field})$$

is conjugate to one of the following subgroups:

$S^1 = \{z \in C;\ |z| = 1\}$, the unit circle in the complex field C;
$NS^1 = \{z,\ zj;\ z \in S^1\}$, the normalizer of S^1 in S^3;
$Z_n = \{z \in S^1;\ z^n = 1\}$, the cyclic group of order n ($\geqq 1$);
$D^*(4m) = \{z,\ zj;\ z \in Z_{2m}\} = \eta_2^{-1}(D(2m))$, the binary dihedral group of order $4m$ ($\geqq 8$);
$T^* = \eta_2^{-1}(T)$, $O^* = \eta_2^{-1}(O)$ and $I^* = \eta_2^{-1}(I)$, the binary tetrahedral, octahedral and icosahedral groups of order 24, 48 and 120, respectively.

Here, η_2: $S^3 \to SO(3)$ is the double covering defined by

$$\eta_2(q)p = qpq^{-1} \qquad (q \in S^3,\ p \text{ is a pure quaternion}),$$

and $D(2m)$ is the dihedral group of order $2m$ and T, O and I are the tetrahedral, octahedral and icosahedral groups.

For an S^3-manifold M, we denote by (H) its type of principal isotropy subgroups, and consider the following two cases:

(a) *Every isotropy subgroup is principal.*

(b) *There exists a non-principal isotropy subgroup* $K \supsetneqq H$.

Unless otherwise stated, we consider S^3/H as the S^3-manifold with the action η_1, $\eta_1(q)[p] = [qp]$. Also, for any S^3-manifold M_1 and any manifold N, we consider $M_1 \times N$ as the S^3-manifold acting S^3 trivially on N.

Then, closed S^3-manifolds are classified up to equivariant diffeomorphisms by the following theorems.