On Higher Coassociativity

Shiroshi SAITO

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Introduction

Co-H-spaces are defined as generalizations of suspended spaces, and, to certain extent, they have dual properties of H-spaces which are considered as generalizations of loop spaces. For H-spaces the so-called Sugawara-Stasheff's sequence of fibrations plays an essential rôle, however, for co-H-spaces we have no such ones. On the other hand, as Ganea pointed out, the coretraction γ for the evaluation map ε seems to be important for co-H-spaces. The purpose of the present paper is to define A'_n-structures which are formal dual of Stasheff's A_n-form and some relevant notions, e. g., A'_n-maps and (weak-) homotopy-coalgebras, and then to consider how γ relates to these notions.

In §1, we give the preliminary definitions and results concerning co-H-spaces and the coretraction γ. In §§2–3, we give the definitions of A'_n-spaces and A'_n-maps and some of their properties. In §4, we define a generalized Hopf-homomorphism $H(f)$ of a map f of A'_2-spaces whose vanishing is equivalent to f being a q-A'_2-map.

Now, our main results are as follows.

THEOREM 5.7. *An A'_3-cogroup X is an s-A'_4-cogroup if and only if the corresponding coretraction γ is a q-A'_3-map.*

THEOREM 6.4. *If X is a simply-connected coalgebra of finite dimension, then X has a homotopy-type of a suspended space.*

THEOREM 6.20. *Let X be an s-A'_4-cogroup such that the corresponding γ is an A'_3-map, then X is a weak homotopy coalgebra of order 3.*

Our method is very elementary-homotopical, and the most difficulties arise from the fact that we must construct the (s-)homotopy of (s-)homotopies.

§1. Preliminaries

In this section, we shall state preliminary facts which will be necessary in the subsequent sections. Throughout present paper, if otherwise not mentioned,