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Stability of Difference Schemes for Nonsymmetric Linear Hyperbolic Systems with Variable Coefficients

Hisayoshi SHINTANI and Kenji TOMOEDA

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1. Introduction

Let us consider the Cauchy problem for a hyperbolic system

$$(1.1)\quad \frac{\partial u}{\partial t}(x, t) = \sum_{j=1}^{n} A_j(x)\frac{\partial u}{\partial x_j}(x, t) \qquad (0 \leqq t \leqq T,\ -\infty < x_j < \infty),$$

$$(1.2)\qquad u(x, 0) = u_0(x), \qquad u_0(x) \in L_2,$$

where $u(x, t)$ and $u_0(x)$ are N-vectors and $A_j(x)$ ($j=1, 2,\dots, n$) are $N \times N$ matrices, and assume that this problem is well posed. For the numerical solution of this problem we consider the difference scheme

$$(1.3)\qquad v(x, t+k) = S_h(x, h)v(x, t) \qquad (0 \leqq t \leqq T,\ -\infty < x_j < \infty),$$

$$(1.4)\qquad v(x, 0) = u_0(x), \qquad k = \lambda h,$$

and study the stability of the scheme in the sense of Lax-Richtmyer, where $S_h(x, h)$ is a difference operator and h is a space mesh width.

The stability of schemes for symmetric hyperbolic systems was studied by Lax [7], Lax and Wendroff [8, 9], Kreiss [5] and Parlett [12] in the case

$$(1.5)\qquad S_h(x, h) = \textstyle\sum_\alpha c_\alpha(x, h)T_h^\alpha,$$

where α is a multi-index, c_α is an $N \times N$ matrix and T_h is the translation operator.

The stability for nonsymmetric hyperbolic systems was treated first by Yamaguti and Nogi [20]. They defined a family of bounded linear operators in L_2 associated with an $N \times N$ matrix $k(x, \omega)$ which is homogeneous of degree zero in ω, is independent of x for $|x| \geqq R$ ($R>0$) and belongs to $C^\infty(R_x^n \times (R_\omega^n - \{0\}))$. They studied the properties of the algebra of such families and applied the results to the investigation of the stability of Friedrichs' scheme under the assumption: The system (1.1) is regularly hyperbolic and $A_j(x)$ ($j=1, 2,\dots, n$) are independent of x for $|x| \geqq R$ and belong to C^∞. Under the same assumption, Vaillancourt [16, 17] obtained an improved stability condition for Friedrichs' scheme and a condition for the modified Lax-Wendroff scheme; Kametaka [4]