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On the Radial Limits of Riesz Potentials at Infinity

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1. Introduction

In this paper, we shall study the limits of potentials on R^n along rays issuing from the origin. It is known that if U_2^μ is the Newtonian potential of a measure μ with finite energy, then $\lim_{r\to\infty} U_2^\mu(r\xi)=0$ for a.e. ξ with $|\xi|=1$ (see N. S. Landkof [2; Theorem 1.21]). We shall deal with the Riesz potential U_α^μ of order α, $0<\alpha<n$, of a measure μ whose energy may not be finite, and give an improvement of the above result (Theorem 1).

We shall then consider the functions of the form

$$F(x) = \int |x-y|^{\alpha-n}|y|^{\beta/p}f(y)dy,$$

where $\alpha>0$, $\beta\geqq 0$, $p>1$, $\alpha p+\beta<n$ and $f\in L^p(R^n)$. In special cases, e.g. in the case where $\alpha=1$, $\beta=0$ and $1<p<n$, *M.* Ohtsuka showed that $\lim_{r\to\infty}F(r\xi)=0$ for a.e. ξ with $|\xi|=1$ ([5; Theorems 9.6 and 9.12, Example 1 given after Theorem 3.21]). This result will be improved in Theorem 2.

Finally we shall be concerned with locally p-precise functions on R^n. We say that a function u is locally p-precise on R^n if u is p-precise on any bounded open set in R^n; for p-precise functions, see [7]. We also refer to [5; Chap. IV]. Let $1<p<n$ and u be a locally p-precise function on R^n such that

$$\int |\operatorname{grad} u|^p|x|^{-\beta}dx < \infty$$

for some non-negative number β smaller than $n-p$. Then we shall show in Theorem 3 that there are a constant c and a set $E\subset\Gamma=\{\xi\in R^n;\ |\xi|=1\}$ such that

$$\lim_{r\to\infty} u(r\xi) = c \qquad \text{if} \quad \xi\in\Gamma-E$$

and

$$C_p(E) = 0 \qquad \text{if} \quad p\leqq 2,$$

$$C_{p-\varepsilon}(E) = 0 \qquad \text{for any } \varepsilon \text{ with } \quad 0<\varepsilon<p \quad \text{if} \quad p>2,$$

where $C_\gamma(E)$ is the Riesz capacity of E of order γ. If, in addition, u is a Riesz potential of a non-negative measure with finite energy, then $c=0$ (cf. [5; Theorem