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On the Existence of Boundary Values of p-Precise Functions

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1. Introduction and statement of results

Let R^n be the n-dimensional Euclidean space ($n \geqq 2$). We use the notation:

$$x = (x', x_n) \in R^{n-1} \times R^1,$$

$$R_+^n = \{x = (x', x_n) \in R^n;\ x_n > 0\},$$

$$R_0^n = \{x = (x', x_n) \in R^n;\ x_n = 0\}.$$

Throughout this paper let $1 < p < \infty$. We say that a function u is locally p-precise on an open set $G \subset R^n$ if u is p-precise on any relatively compact open subset of G; for p-precise functions, see [10]. For a real number α, we consider a locally p-precise function u on R_+^n such that

$$\iint_{R_+^n} |\text{grad}\, u|^p x_n^\alpha dx' dx_n < \infty . \tag{1}$$

In case $\alpha \geqq 0$ and $1+\alpha < p < n+\alpha$, we have already discussed the existence of $\lim u(x', x_n)$ as $x_n \downarrow 0$ ([6]). In the present paper, we shall discuss it in more general cases. We denote by C_ℓ ($0 < \ell < n$) the Riesz capacity of order ℓ (which refers to the kernel $|x|^{\ell-n}$), by C_n the logarithmic capacity and by $B_{\ell,p}$ ($0 < \ell < \infty$) the Bessel capacity of index (ℓ, p) (cf. [4]).

First we state

THEOREM 1. *Let u be a locally p-precise function on R_+^n satisfying* (1). *Then there is a Borel set $E \subset R_0^n$ such that*

$B_{1,p}(E) = 0$ *if* $\alpha \leqq 0$,

$C_{p-\alpha}(E) = 0$ *if* $\alpha > 0$ *and* $1+\alpha < p \leqq 2$,

$C_{p-\alpha-\varepsilon}(E) = 0$ *for any* ε *with* $0 < \varepsilon < p-\alpha$ *if* $\alpha > 0$, $p > 2$ *and* $1+\alpha < p \leqq n+\alpha$,

E *is empty* *if* $0 < \alpha < p-n$