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Principal Oriented Bordism Modules of Finite Subgroups of S^3

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Introduction

The principal oriented bordism module $\Omega_*(G)$ of a compact Lie group G is defined to be the module of all equivariant bordism classes of closed principal oriented (smooth) G-manifolds, and is a module over the oriented bordism ring Ω_* of R. Thom (cf. [2]).

Let G be a finite subgroup of the unit sphere S^3 in the quaternion field $\boldsymbol{H}$. Then, it is well known that G is a cyclic group Z_m, a binary dihedral group $D^*(4m)$ or a binary polyhedral group T^*, O^*, or I^* (cf. (1.1)).

The Ω_*-module structure of $\Omega_*(G)$ is determined by P. E. Conner and E. E. Floyd [2, Ch. VII] for $G=Z_{p^k}$ (p: odd prime), and by K. Shibata [7, §§1–4] for $G=Z_2$. Also, it is proved by N. Hassani [3] that there is an isomorphism $\Omega_*(Z_{mm'})=\Omega_*(Z_m)\otimes_{\Omega_*}\Omega_*(Z_{m'})$ if m and m' are relatively prime.

Furthermore, in the recent papers K1 and K2, we have determined $\Omega_*(G)$ for $G=Z_{2^k}$, $k\geqq 2$ (K1-Theorem 2.18), and for $G=H_m=D^*(2^{m+1})$, $m\geqq 2$ (K2-Theorem 8.12). As a continuation to these papers, we study in this paper the Ω_*-module structures of $\Omega_*(G)$ for the remaining finite subgroups G of S^3, that is,

$$G = D^*(2^{m+1}t) \ (t\colon \text{odd} \geqq 3,\ m \geqq 1), \quad T^*, \quad O^* \quad \text{and} \quad I^*.$$

Our results are stated in Theorems 4.8, 5.9, 6.10 and 7.8 as follows:

$$\tilde{\Omega}_*(D^*(2^{m+1}t)) = D\tilde{\Omega}_*(D^*(2^{m+1})) \oplus D'\mathfrak{Z}_{t,1},$$

$$\tilde{\Omega}_*(T^*) = T(\mathfrak{L}_2 \oplus \mathfrak{W}_2) \oplus T'\tilde{\Omega}_*(Z_3),$$

$$\tilde{\Omega}_*(O^*) = \mathfrak{O} \oplus O(\mathfrak{W}_2 \oplus \mathfrak{Q}_2),$$

$$\tilde{\Omega}_*(I^*) = \mathfrak{I} \oplus I_4\mathfrak{G}_{2,1} \oplus I_3\mathfrak{Z}_{3,1} \oplus I_5\mathfrak{Z}_{5,1}.$$

Here

$$\tilde{\Omega}_*(D^*(2^{m+1})) = \mathfrak{L}_m \oplus \mathfrak{W}_m \oplus \mathfrak{Q}_m \oplus \mathfrak{Q}'_m \quad \text{(cf. K2-Theorem 8.12)},$$

$$\tilde{\Omega}_*(Z_t) = \mathfrak{Z}_{t,0} \oplus \mathfrak{Z}_{t,1} \quad \text{(cf. Theorem 3.8)},$$