HIROSHIMA MATH. J.
7 (1977), 61–70

Volterra Integral Equations as Functional Differential Equations on Infinite Intervals

G. F. WEBB

(Received April 28, 1976)

1. Introduction

Our objective is to treat the nonlinear functional differential equation

$$\text{(1.1)} \qquad \dot{x}(t) = f(t, x(t)) + F(t, x_t), \qquad t \geq s \geq 0$$
$$x(s) = h \in H, \qquad x_s = \phi \in L^p(-r, 0; H)$$

by means of the theory of nonlinear evolutions in Banach spaces. The notation of (1.1) means that H is a Hilbert space, $1 \leq p < \infty$, $0 < r \leq \infty$, $x: (s-r, \infty) \to H$, $f: [0, \infty) \times H \to H$, $F: [0, \infty) \times L^p(-r, 0; H) \to H$, and $x_t \in L^p(-r, 0; H)$ is defined by $x_t(\theta) = x(t+\theta)$ for $-r < \theta < 0$. The equation (1.1) may be formulated as the abstract ordinary differential equation

$$\text{(1.2)} \qquad du(t)/dt = -Au(t) - B(t, u(t)), \qquad t \geq s \geq 0$$
$$u(s) = x \in X$$

in the Banach space $X \stackrel{\text{def}}{=} L^p(-r, 0; H) \times H$. The notation of (1.2) means u: $[s, \infty) \to X$, $A: X \to X$ such that $-A$ is the infinitesimal generator of a strongly continuous semigroup of linear operators in X, and $B: [0, \infty) \times X \to X$ such that $B(t, \cdot)$ is nonlinear. By converting (1.1) to the form (1.2), we will be able to take advantage of the extensive theory which has been developed in recent years for nonlinear evolution equations of the form (1.2). The special semi-linear form of (1.2) will allow us to state explicitly the relationship between the solutions of (1.1) and (1.2).

There is a growing literature associated with the treatment of functional differential equations as abstract ordinary differential equations in function spaces, and some recent papers on this subject are listed in our references. Our work continues the investigations of [17], [18], and [19], where (1.1) was treated with F independent of t and $r < \infty$. In the present study we will allow the case that $r = \infty$, so that we may treat nonlinear Volterra integral equations of the form

This research was supported in part by the National Science Foundation Grant NSF–MPS 75–06332.