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Vanishing Nonoscillations of Lienard Type Retarded Equations

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1. Introduction

Our purpose in this paper is to study the asymptotic nature of nonoscillatory solutions of the equation

$$(r(t)x'(t))' + p(t)x'(t) + q(t)x(t) + a(t)h(x(g(t))) = f(t) \tag{1}$$

under the assumptions:

(i) $r(t)$, $p(t)$ and $g(t)$ are nonnegative, real valued and continuous on the whole real line R;

(ii) $a(t)$, $f(t)$, $q(t)$: $R \to R$ and continuous;

(iii) $r(t)$, $p(t)$ and $g(t)$ are $C'[A, \infty)$ for some $A > 0$;

(iv) $r'(t) \geq 0$, $0 < g'(t) \leq S$ for some $S > 0$, $r(t) \geq K > 0$ and bounded; let $R(t) = \int^t 1/r(s)ds$;

(v) h: $R \to R$, increasing, $\operatorname{sign}(h(t)) = \operatorname{sign} t$, $h(-t) = -h(t)$, and if $t \to 0$, then $h(t) \to 0$, $h(t)/t$ is bounded;

(vi) $g(t) \leq t$ and $g(t) \to \infty$ as $t \to \infty$.

A function $y(t) \in C[A, \infty)$ is said to be nonoscillatory, if it eventually assumes a constant sign for arbitrarily large values of t; otherwise it is called oscillatory.

The existence of the continuously extendable solutions of equation (1) will be taken for granted. From here on the term "solution" applies only to such solutions on $[A, \infty)$.

Recently T. Kusano and H. Onose [4] studied the equation

$$(r(t)y'(t))' + a(t)h(y(g(t))) = f(t) \tag{2}$$

under practically similar assumptions and showed that bounded nonoscillatory solutions of (2) would approach to zero if

$$\int^{\infty} R(t)a^{-}(t)dt < \infty, \qquad \int^{\infty} R(t)a^{+}(t)dt = \infty$$

and $\int^{\infty} R(t)|f(t)|dt < \infty$. It will be shown in this manuscript that these conditions are strong enough to cause all nonoscillatory solutions of (2) to approach zero. In the process known results of Hammett [3], Grimmer [2], Londen [5], and