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A Note on Generalized Factorial Series Expansions

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§1. Introduction

As is well known, actual solutions of linear ordinary differential equations in a neighborhood of an irregular singular point are characterized and asymptotically represented by formal power series solutions which are in general divergent. It may be more desirable to obtain the convergent representation of solutions in a neighborhood of an irregular singular point in computing the exact value of a solution. For this objective, J. Horn [1, 2], W. J. Trjitzinsky [12] and H. L. Turrittin [14] attempted to sum formal power series solutions by means of the so-called Borel exponential summation and obtained convergent generalized factorial series expansions of actual solutions near an irregular singular point in some cases. In particular, H. L. Turrittin attacked this problem of summation for systems of linear differential equations believing that all formal power series solutions could be summed in every case. Although a considerable progress was made, he did not succeed in summing formal power series solutions in all cases in his paper [14]. See also [15].

The method of obtaining convergent generalized factorial series expansions of actual solutions near an irregular singular point is due to the decomposition of an original system of linear differential equations into a sum of a certain number of nonhomogeneous systems of linear differential equations whose solutions are expressed in terms of Laplace integrals.

We here consider a system of linear differential equations of the form

$$\tau\frac{dX}{d\tau} = \tau^{g}A(\tau)X, \tag{1.1}$$

where the matrix $A(\tau)$ is holomorphic at $\tau=\infty$, i.e., it permits a convergent power series expansion

$$A(\tau) = \sum_{m=0}^{\infty} A_m\tau^{-m} \tag{1.2}$$

for sufficiently large values of τ.
If we then put

$$X(\tau) = \sum_{l=0}^{q-1}\{C_l\tau^{-l} + \tau^{-l}Z_l(\xi)\}\tau^{-h} \qquad (\xi = \tau^{q}) \tag{1.3}$$