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On *Dedekind Domains

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In this paper a ring is always a commutative ring with a unit and a graded ring is a ring with a grading of Z-type. Let A be a graded domain. Then the set of non-zero graded fractional ideals of A becomes a monoid naturally. We shall say that A is a Dedekind domain in the category of graded rings when the monoid is a group; it is not so difficult to show that A is a Dedekind domain in the category of graded rings if and only if the global dimension of A in the category of graded A-modules is less than or equal to 1.

The main purpose of this paper is to determine the structure of graded domains of global dimension 1, or equivalently to determine the structure of Dedekind domains in the category of graded rings. To do this we shall introduce a notion of exceptional primes, which plays an important role in this paper. As a main result we shall show that, in case exceptional primes do not appear, there is a 1:1 correspondence between the set of isomorphism classes and the class group of the Dedekind domain consisting of homogeneous elements of degree zero.

1. Graded rings

Let $A=\oplus A_n$, $n\in Z$, be a graded ring. We let $h(A)$ denote the set of homogeneous elements of A. Given a homogeneous element x, $\deg(x)$ stands for the degree of x. Let $\mathrm{Gr}(A)$ be the category of graded modules over A. A morphism f of M to N is an A-homomorphism of M to N such that $f(M_n)\subset N_n$ for every n, where $M=\oplus M_n$, $n\in Z$, and $N=\oplus N_n$, $n\in Z$. An asterisk (*) means 'graded', 'homogeneous' or 'in $\mathrm{Gr}(A)$'. For example, a *module is a graded module, and a *ideal is a homogeneous ideal, and so on. If every *ideal of A is finitely generated, then we say A is *noetherian. When A is *noetherian, A_0 is noetherian and moreover the underlying ring of A is noetherian ([3], p. 306). For an ideal $\mathfrak{a}$ of a *ring, we denote by $\mathfrak{a}^*$ the *ideal generated by all the *elements of $\mathfrak{a}$. If $\mathfrak{a}$ is a prime ideal, then $\mathfrak{a}^*$ is also a prime *ideal.

PROPOSITION 1.1. *Let A be a *domain. If the Jacobson radical is not zero, then $A=A_0$.*

PROOF. Suppose that there exists a non-zero *element b with $\deg(b)\neq 0$.