

Some Results on Hopf Algebras Attached to Group Schemes

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It is known that some roles played by Lie algebras attached to algebraic groups over a field of characteristic zero are played instead by Hopf algebras attached to them in the case of positive characteristic. This is essentially due to the fact that the enveloping algebra of the Lie algebra attached to an algebraic group over a field of positive characteristic is a proper subalgebra of the Hopf algebra attached to it in contrast to the case of characteristic zero, where the Hopf algebra attached to an algebraic group coincides with the enveloping algebra of the Lie algebra attached to it. Hence there arises a motivation to study Hopf algebras attached to group schemes over a field of arbitrary characteristic. In other words if we want to develop an infinitesimal theory of group schemes over a field of arbitrary characteristic, it would be natural to treat rather Hopf algebras than Lie algebras.

The purpose of this paper is to give a theory of Hopf algebras attached to group schemes over an algebraically closed field of arbitrary characteristic, which corresponds to the theory of Lie algebras attached to algebraic groups over a field of characteristic zero developed by C. Chevalley and A. Borel in their books [2] and [1] respectively. In particular we shall show some interesting results on algebraic Hopf subalgebras in connection with adjoint representations of group schemes. Although there are some results on this subject obtained already by J. Dieudonné and M. Takeuchi in their papers [3] and [11] respectively, it seems to the author that their results do not cover the whole which would correspond to the results on Lie algebras in characteristic zero case. For example there is no result on joins of connected group subschemes which are not necessarily reduced.

In § 1 we recall the definition and some properties of group schemes, and then we define Hopf algebras attached to group schemes and other notions necessary in the later sections. The notion and basic properties of h -inverses of Hopf subalgebras by a Hopf algebra homomorphism will be given in § 2. We shall show some basic results on algebraic Hopf subalgebras in § 3. In particular we define the algebraic hull of a Hopf subalgebra of the Hopf algebra attached to

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