HIROSHIMA MATH. J.
7 (1977), 787–812

Stability of Difference Schemes for Nonsymmetric Linear Hyperbolic Systems

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(Received May 18, 1977)

1. Introduction

Let us consider the Cauchy problem for a hyperbolic system

$$(1.1)\quad \frac{\partial u}{\partial t}(x, t) = \sum_{j=1}^{n} A_j(x, t)\frac{\partial u}{\partial x_j}(x, t) \qquad (0 \leqq t \leqq T, -\infty < x_j < \infty),$$

$$(1.2)\qquad u(x, 0) = u_0(x), \quad u_0(x) \in L_2,$$

where $u(x, t)$ and $u_0(x)$ are N-vectors and $A_j(x, t)$ $(j=1, 2,\dots, n)$ are $N\times N$ matrices, and assume that this problem is well posed. For the numerical solution of this problem we consider the following difference scheme:

$$(1.3)\qquad v(x, t+k) = S_h(t, h)v(x, t) \qquad (0 \leqq t \leqq T, -\infty < x_j < \infty),$$

$$(1.4)\qquad v(x, 0) = u_0(x), \quad k = \lambda h \qquad (\lambda > 0),$$

where $S_h(t, \mu)$ is a sum of products of operators of the form $\sum_\alpha c_\alpha(x, t, \mu)T_h^\alpha$ $(\mu \geqq 0)$, α is a multi-index, $c_\alpha(x, t, \mu)$ is an $N\times N$ matrix, T_h is the translation operator and h is a space mesh width.

In our previous paper [5] we treated the case where $A_j(x, t)$ $(j=1, 2,\dots, n)$ are independent of t, and obtained sufficient conditions for L_2-stability of the scheme (1.3). In this paper we extend the results to the system (1.1) that satisfies the following conditions: Eigenvalues of $A(x, t, \xi) = \sum_{j=1}^{n} A_j(x, t)\xi_j/|\xi|$ $(\xi \neq 0)$ are all real and their multiplicities are independent of x, t and ξ; elementary divisors of $A(x, t, \xi)$ are all linear; there exists a positive constant δ such that

$$|\lambda_i(x, t, \xi) - \lambda_j(x, t, \xi)| \geqq \delta \qquad (i \neq j;\ i, j = 1, 2,\dots, s),$$

where $\lambda_i(x, t, \xi)$ $(i=1, 2,\dots, s)$ are all the distinct eigenvalues of $A(x, t, \xi)$.

Our proof of stability is based on the following result: The scheme (1.3) is stable if $S_h(t, h)$ and $S_h(t, 0)$ are the families of bounded linear operators in L_2 and if there exist positive constants c_j $(j=0, 1, 2)$ and a norm $|||\cdot|||_t$ which depends on t and is equivalent to the L_2-norm such that

$$(1.5)\qquad |||u|||_{t+k} \leqq (1 + c_0 k)\,|||u|||_t \qquad (t + k \leqq T),$$