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Some Remarks on Representations of p-adic Chevalley Groups

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Introduction

Let F be a p-adic field, and let $\mathfrak{O}$ and $\mathfrak{P}$ be the ring of integers and the maximal ideal of $\mathfrak{O}$ respectively. F. I. Mautner [4] first constructed square-integrable irreducible unitary representations of $PGL_2(F)$ which are induced by irreducible representations of a certain maximal compact subgroup. In [5], J. A. Shalika carried it out for $SL_2(F)$ by a different method. Independently, in [6] and [7], T. Shintani extended Mautner's results to a sort of special linear group of rank n. Recently, in [2] and [3], P. Gérardin extended their results to reductive)p-adic groups whose semi-simple parts are simply connected.

In this paper, we extend the former results of [7], which are not covered by Gérardin's results, to general p-adic Chevalley groups. The contents of this paper are as follows. Let $G(\mathbf{z})$ be a Chevalley group over the ring of all rational integers $\mathbf{z}$. Then we have a p-adic Chevalley group $G(F)$ and its maximal compact subgroup $G(\mathfrak{O})$ by base changes. In § 1, we give preliminaries on the structures of p-adic Chevalley groups after [3]. In § 2, we prepare certain lemma about induced representations of finite groups. In § 3, we show that continuous irreducible unitary representations of $G(\mathfrak{O})$, which do not come from representations of $G(\mathfrak{O}/\mathfrak{P})$, are induced by certain irreducible representations of certain subgroups of $G(\mathfrak{O})$ (Theorem 1). In § 4, when we let ν be a continuous irreducible unitary representation of $G(\mathfrak{O})$ which does not come from a representation of $G(\mathfrak{O}/\mathfrak{P})$, we obtain a sufficient condition for $\mathrm{Ind}_{G(\mathfrak{O})}^{G(F)}\nu$ to be square-integrable.

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NOTATIONS: (i) Let F be a non-archimedean local field, and let $\mathfrak{O}$, $\mathfrak{P}$ and π be the ring of integers of F, the maximal ideal of $\mathfrak{O}$, and a prime element of F, respectively. Let p be the characteristic of the finite field $\mathfrak{O}/\mathfrak{P}$.

(ii) For a ring R, we denote by $M(n_1, n_2, R)$ the set of n_1 by n_2 matrices with coefficients in R. We put $M(n, R)=M(n, n, R)$.

(iii) For each positive integer m, we denote by ψ_m the reduction modulo $\mathfrak{P}^m$: $\mathfrak{O}\to\mathfrak{O}/\mathfrak{P}^m$. For integers $n\geqq m\geqq 1$, we denote by the same symbol ψ_m the reduction modulo $\mathfrak{P}^m$: $\mathfrak{O}/\mathfrak{P}^n\to\mathfrak{O}/\mathfrak{P}^m$.